Hands-on Distributional Semantics

Part 2: The parameters of a DSM

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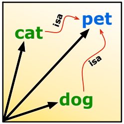
http://wordspace.collocations.de/doku.php/course:esslli2021:start



Outline

General definition of DSMs

A **distributional semantic model** (DSM) is a scaled and/or transformed co-occurrence matrix **M**, such that each row **x** represents the distribution of a target term across contexts.

	get	see	use	hear	eat	kill
knife	0.027	-0.024	0.206	-0.022	-0.044	-0.042
cat	0.031	0.143	-0.243	-0.015	-0.009	0.131
dog	-0.026	0.021	-0.212	0.064	0.013	0.014
boat	-0.022	0.009	-0.044	-0.040	-0.074	-0.042
cup	-0.014	-0.173	-0.249	-0.099	-0.119	-0.042
pig	-0.069	0.094	-0.158	0.000	0.094	0.265
banana	0.047	-0.139	-0.104	-0.022	0.267	-0.042

Term = word, lemma, phrase, morpheme, word pair, . . .

General definition of DSMs

Mathematical notation:

- $k \times n$ co-occurrence matrix $\mathbf{M} \in \mathbb{R}^{k \times n}$ (example: 7×6)
  - k rows = **target** terms
  - n columns = **features** or other **dimensions**

$$\mathbf{M} = \begin{bmatrix} m_{11} & m_{12} & \cdots & m_{1n} \\ m_{21} & m_{22} & \cdots & m_{2n} \\ \vdots & \vdots & & \vdots \\ m_{k1} & m_{k2} & \cdots & m_{kn} \end{bmatrix}$$

- distribution vector $\mathbf{m}_i = i$-th row of $\mathbf{M}$, e.g. $\mathbf{m}_3 = \mathbf{m}_{\text{dog}} \in \mathbb{R}^n$
- components $\mathbf{m}_i = (m_{i1}, m_{i2}, \ldots, m_{in})$ = features of i-th term:

$$\begin{aligned} \mathbf{m}_3 &= (-0.026, 0.021, -0.212, 0.064, 0.013, 0.014) \\ &= (m_{31}, m_{32}, m_{33}, m_{34}, m_{35}, m_{36}) \end{aligned}$$

Term-term matrix

Term-term matrix records co-occurrence frequencies with feature terms for each target term

☞ $\mathbf{m}_{\text{dog}}$ = collocational profile of *dog* (≈ word sketch)

$$\mathbf{M} = \begin{bmatrix} \cdots & \mathbf{m}_1 & \cdots \\ \cdots & \mathbf{m}_2 & \cdots \\ & \vdots & \\ & \vdots & \\ \cdots & \mathbf{m}_k & \cdots \end{bmatrix}$$

	breed	tail	feed	kill	important	explain	likely
cat	83	17	7	37	–	1	–
dog	561	13	30	60	1	2	4
animal	42	10	109	134	13	5	5
time	19	9	29	117	81	34	109
reason	1	–	2	14	68	140	47
cause	–	1	–	4	55	34	55
effect	–	–	1	6	60	35	17

```
> TT <- DSM_TermTerm
> head(TT, Inf)  # extract full co-oc matrix from DSM object
```

Term-context matrix

Term-context matrix records frequency of term in each individual context unit (e.g. document, tweet, encyclopaedia article)

☞ $\mathbf{f}_{\text{dog}}$ = texts related to or mentioning dogs

$$\mathbf{F} = \begin{bmatrix} \cdots & \mathbf{f}_1 & \cdots \\ \cdots & \mathbf{f}_2 & \cdots \\ & \vdots & \\ & \vdots & \\ \cdots & \mathbf{f}_k & \cdots \end{bmatrix}$$

	Felidae	Pet	Feral	Bloat	Philosophy	Kant	Back pain
cat	10	10	7	–	–	–	–
dog	–	10	4	11	–	–	–
animal	2	15	10	2	–	–	–
time	1	–	–	–	2	1	–
reason	–	1	–	–	1	4	1
cause	–	–	–	2	1	2	6
effect	–	–	–	1	–	1	–

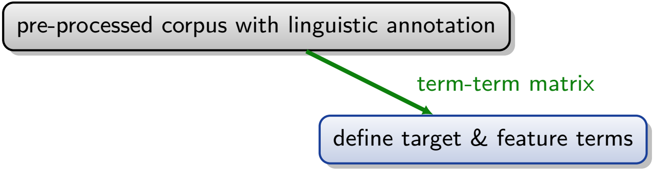
```
> TC <- DSM_TermContext
> head(TC, Inf)
```

Outline

Building a distributional model

pre-processed corpus with linguistic annotation

Building a distributional model



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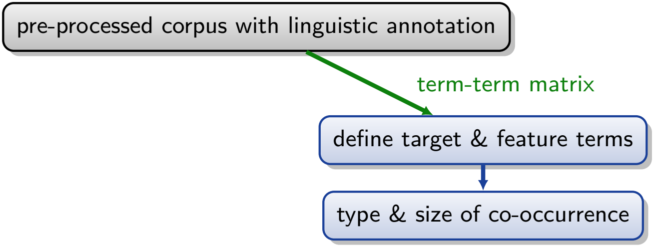

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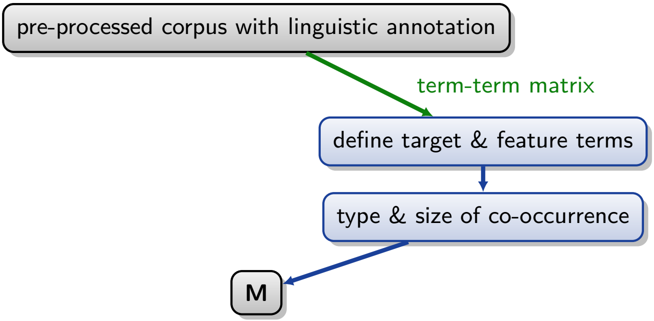

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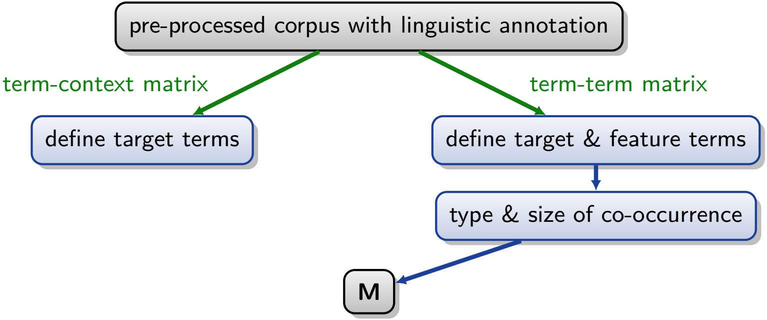

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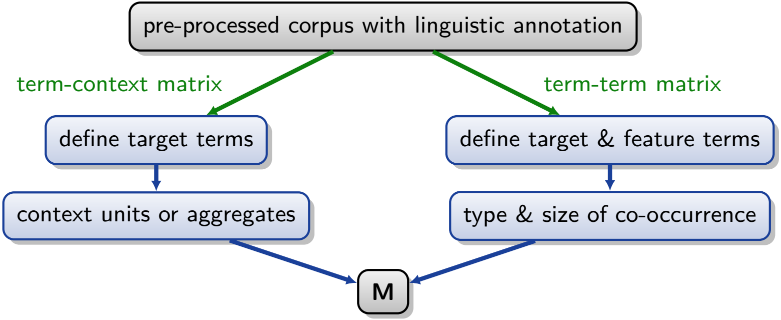

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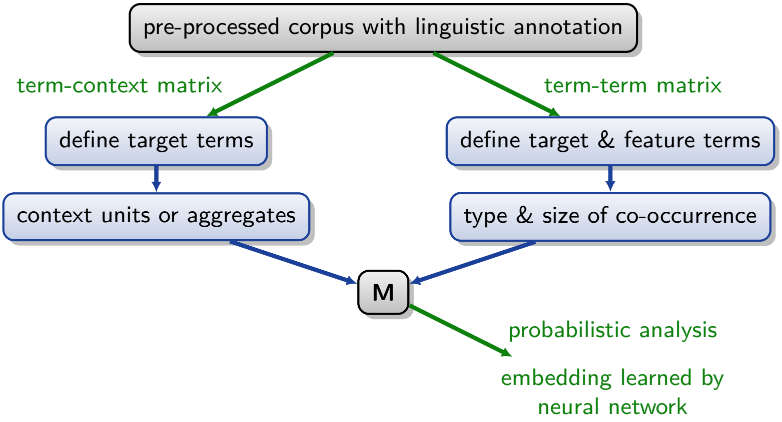

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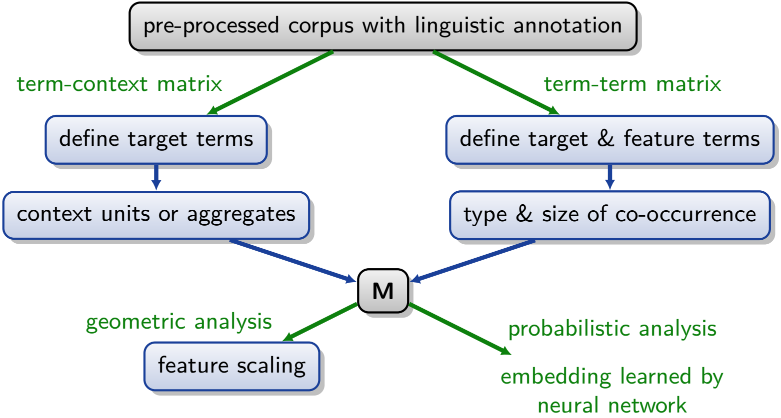

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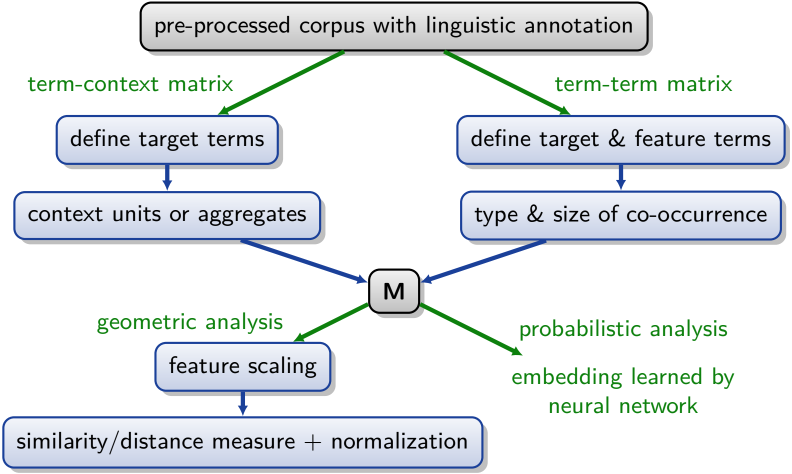

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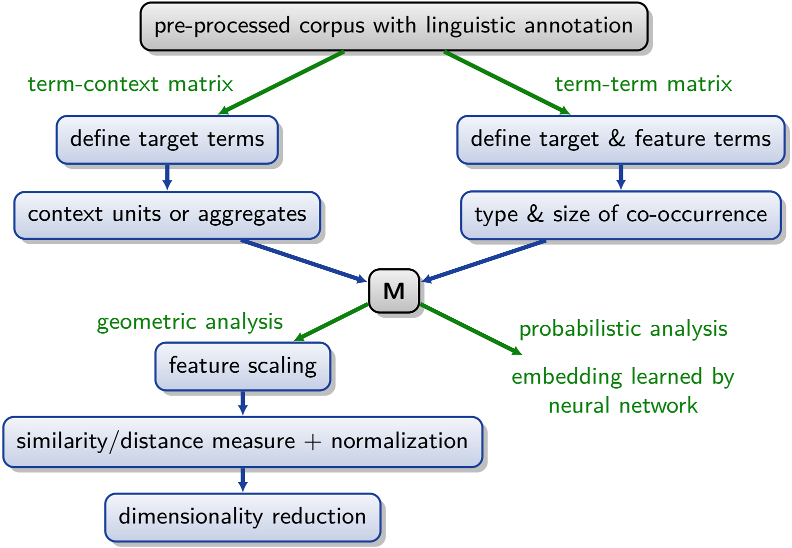

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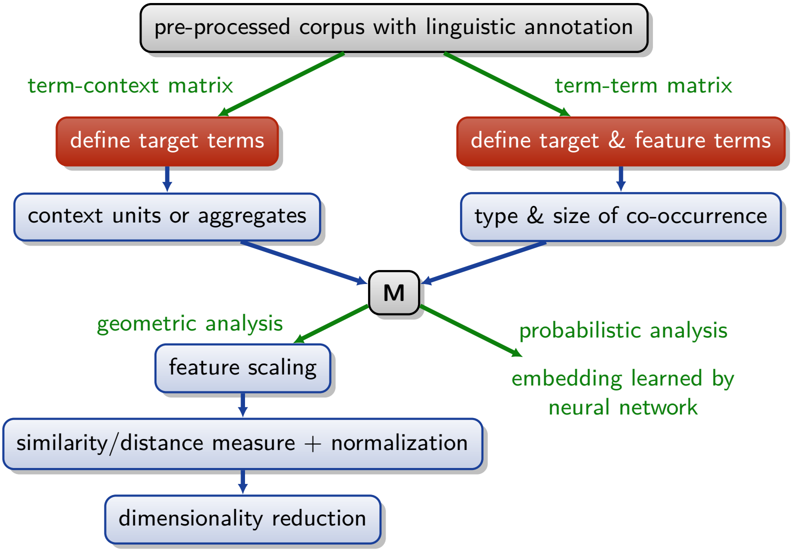

Definition of target and feature terms

- Choice of linguistic unit (targets $\neq$ features)
  - words
  - bigrams, trigrams, ...
  - multiword units, named entities, phrases, ...
  - morphemes
  - word pairs (☞ analogy tasks)

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- Mapping to target/feature terms (➜ linguistic annotation)
  - word forms (minimally requires tokenisation)
  - often lemmatisation or stemming to reduce data sparseness: *go, goes, went, gone, going* $\rightarrow$ *go*
  - POS disambiguation (*light*/N vs. *light*/A vs. *light*/V)
  - word sense disambiguation (*bank*$_{\text{river}}$ vs. *bank*$_{\text{finance}}$)
  - abstraction: POS tags (or *n*-grams of POS tags) as features

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☞ What is the effect of these choices?

Effects of term mapping

Nearest neighbours of *walk* (BNC)

word forms	lemmatised + POS
stroll	hurry
walking	stroll
walked	stride
go	trudge
path	amble
drive	wander
ride	walk (noun)
wander	walking
sprinted	retrace
sauntered	scuttle

http://clic.cimec.unitn.it/infomap-query/

Effects of term mapping

Nearest neighbours of *arrivare* (Repubblica)

word forms	lemmatised + POS
giungere	giungere
raggiungere	aspettare
arrivi	attendere
raggiungimento	arrivo (noun)
raggiunto	ricevere
trovare	accontentare
raggiunge	approdare
arrivasse	pervenire
arriverà	venire
concludere	piombare

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Selection of target and feature terms

- Full-vocabulary models are often unmanageable
  - 762,424 distinct word forms in BNC, 605,910 lemmata
  - large Web corpora have $>$ 10 million distinct word forms
  - low-frequency targets (and features) are not reliable ("noisy")

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  - corpus frequency $f \geq F_{\text{min}}$ or n_w most frequent terms
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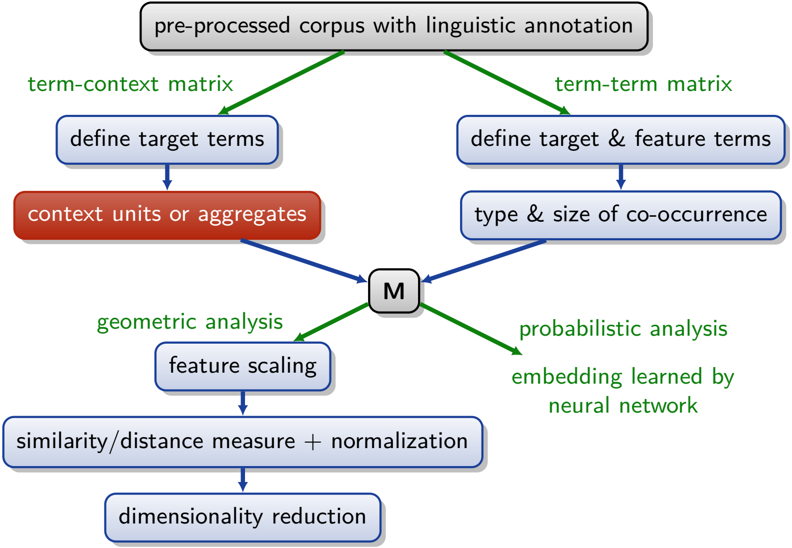
  - alternatives: entropy H or chi-squared statistic X^2
- Other criteria
  - POS-based filter: no function words, only verbs, nouns, ...
  - general dictionary, words required for particular task, ...

Building a distributional model



Term-context matrix: choice of context unit

- Features are usually **tokens** of the selected context unit, i.e. individual instances of a
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- Similar context tokens can be **aggregated**, e.g.
  - feature = cluster of near-duplicate documents
  - feature = syntactic structure of sentence (ignoring content)
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- Generalization: context **types**
  - e.g. pattern of POS tags around target word
  - e.g. subcategorisation pattern of target verb

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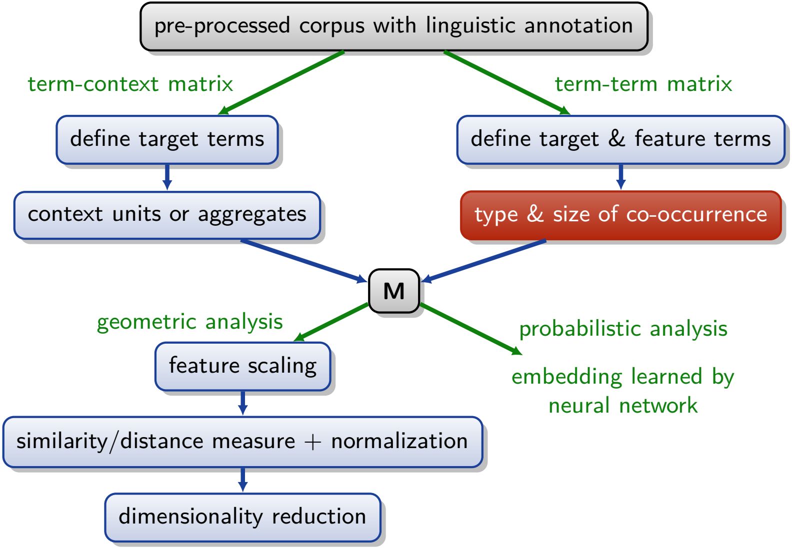

Term-term matrix: definition of co-occurrence context

- Different types of co-occurrence (Evert 2008)
  - **surface context** (word or character window)
  - **textual context** (non-overlapping segments)
  - **syntactic context** (dependency relations)
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☞ What effects do you expect from these choices?

Surface context

Context term occurs within a span of k words around target.

The silhouette of the sun beyond a wide-open bay on the lake; the sun still glitters although evening has arrived in Kuhmo. It's midsummer; the living room has its instruments and other objects in each of its corners. [L3/R3 span, $k = 6$]

Parameters:

- span size (in words or characters)
- symmetric vs. one-sided span
- uniform or "triangular" (distance-based) weighting (don't!)
- spans clamped to sentences or other textual units?

Effect of span size

Nearest neighbours of *dog* (BNC)

2-word span

- cat
- horse
- fox
- pet
- rabbit
- pig
- animal
- mongrel
- sheep
- pigeon

30-word span

- kennel
- puppy
- pet
- bitch
- terrier
- rottweiler
- canine
- cat
- to bark
- Alsatian

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Textual context

Context term is in the same linguistic unit as target.

The silhouette of the sun beyond a wide-open bay on the lake; the sun still glitters although evening has arrived in Kuhmo. It's midsummer; the living room has its instruments and other objects in each of its corners.

Parameters:

- choice of linguistic unit
  - sentence
  - paragraph
  - turn in a conversation
  - Web page
  - tweet

☞ similar to large surface spans, but more self-contained

Syntactic context

Context term is linked to target by a syntactic dependency (e.g. subject, modifier, ...).

The silhouette of the sun beyond a wide-open bay on the lake; the sun still glitters although evening has arrived in Kuhmo. It's midsummer; the living room has its instruments and other objects in each of its corners.

Parameters:

- types of syntactic dependency (Padó & Lapata 2007)
- maximal length of dependency path (1 for direct relation)
- homogeneous data (e.g. only verb-object) vs.
  heterogeneous data (e.g. all children and parents of the verb)

"Knowledge pattern" context

Context term is linked to target by a lexico-syntactic pattern (text mining, cf. Hearst 1992, Pantel & Pennacchiotti 2008, etc.).

In Provence, Van Gogh painted with bright colors such as red and yellow. These colors produce incredible effects on anybody looking at his paintings.

Parameters:

- inventory of lexical patterns
  - lots of research to identify semantically interesting patterns (cf. Almuhareb & Poesio 2004, Veale & Hao 2008, etc.)
- fixed vs. flexible patterns
  - patterns are mined from large corpora and automatically generalised (optional elements, POS tags or semantic classes)

Comparison of co-occurrence contexts

Contexts range from general/implict to specific/explicit:

	features are
textual / large span	from same topic domain

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syntactic (single relation)	attributes (focus on aspect)

Comparison of co-occurrence contexts

Contexts range from general/implict to specific/explicit:

	features are
textual / large span	from same topic domain
small span	collocations
syntactic (single relation)	attributes (focus on aspect)
knowledge pattern	properties

Structured vs. unstructured context

- In **unstructured** models, context specification acts as a **filter**
  - determines whether context token counts as co-occurrence
  - e.g. must be linked by any direct syntactic dependency relation

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- In **unstructered** models, context specification acts as a **filter**
  - determines whether context token counts as co-occurrence
  - e.g. must be linked by any direct syntactic dependency relation
- In **structured** models, feature terms are **subtyped**
  - depending on their position in the context
  - e.g. left vs. right context, type of syntactic relation, etc.

Structured vs. unstructured surface context

A dog bites a man. The man's dog bites a dog. A dog bites a man.

unstructured	bite
dog	4
man	3

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structured	bite-L	bite-R
dog	1	3
man	2	1

Structured vs. unstructured surface context

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➥ data are less sparse (L/R context aggregated)

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structured	bite-L	bite-R
dog	1	3
man	2	1

➥ more sensitive to semantic distinctions

Structured vs. unstructured dependency context

A dog bites a man. The man's dog bites a dog. A dog bites a man.

unstructured	bite
dog	4
man	2

➥ data are less sparse (all syntactic relations aggregated)

A dog bites a man. The man's dog bites a dog. A dog bites a man.

structured	bite-subj	bite-obj
dog	3	1
man	0	2

➥ more sensitive to semantic distinctions

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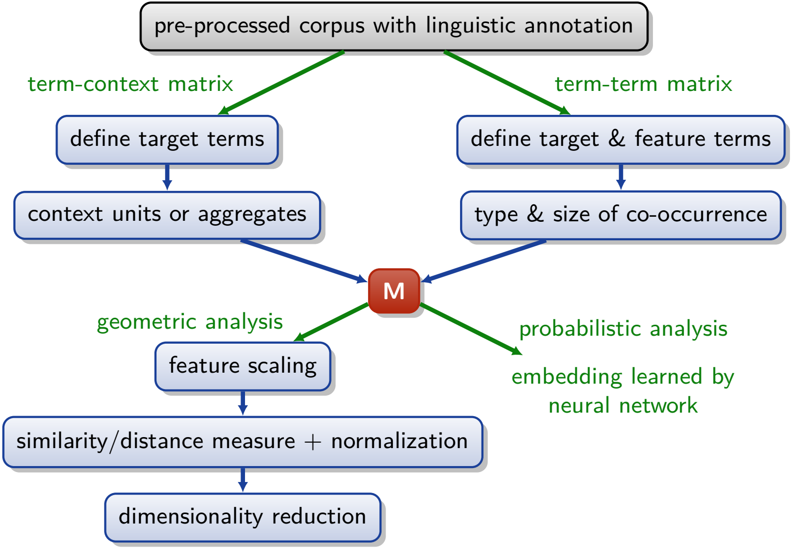

Marginal and expected frequencies

- Matrix of observed co-occurrence frequencies not sufficient

target	feature	O
dog	*small*	855
dog	*domesticated*	29

- Notation
  - O = observed co-occurrence frequency

Marginal and expected frequencies

- Matrix of observed co-occurrence frequencies not sufficient

target	feature	O	R	C
dog	*small*	855	33,338	490,580
dog	*domesticated*	29	33,338	918

- Notation
  - O = observed co-occurrence frequency
  - R = overall frequency of target term = row marginal frequency
  - C = overall frequency of feature = column marginal frequency
  - N = sample size $\approx$ size of corpus

Marginal and expected frequencies

- Matrix of observed co-occurrence frequencies not sufficient

target	feature	O	R	C	E
dog	*small*	855	33,338	490,580	134.34
dog	*domesticated*	29	33,338	918	0.25

- Notation
  - O = observed co-occurrence frequency
  - R = overall frequency of target term = row marginal frequency
  - C = overall frequency of feature = column marginal frequency
  - N = sample size $\approx$ size of corpus
- **Expected** co-occurrence **frequency** (cf. Evert 2008)

$$E = \frac{R \cdot C}{N} \quad \longleftrightarrow \quad O$$

Obtaining marginal frequencies (Evert 2008)

- Term-document matrix
  - R = frequency of target term in corpus
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  - N, R, C can be computed from full co-occurrence matrix $\mathbf{M}$

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- Syntactic co-occurrence
  - # of dependency instances in which target/feature participates
  - N = total number of dependency instances
  - N, R, C can be computed from full co-occurrence matrix $\mathbf{M}$
- Textual co-occurrence
  - R, C, O are "document" frequencies, i.e. number of context units in which target, feature or combination occurs
  - N = total # of context units

Obtaining marginal frequencies (Evert 2008)

- Surface co-occurrence
  - it is quite tricky to obtain fully consistent counts
  - at least correct E for span size k (= # tokens in span)[1]

$$E = k \cdot \frac{R \cdot C}{N}$$

with R, C = individual corpus frequencies and N = corpus size

  - can also be implemented by pre-multiplying $R' = k \cdot R$
  - approach used for all pre-compiled surface DSMs in the course

☞ alternatively, compute marginals and sample size by summing over full co-occurrence matrix (➜ E as above, but inflated N)

[1] NB: shifted PPMI (Levy & Goldberg 2014) corresponds to a post-hoc application of the span size adjustment. It performs worse than PPMI, but paper suggests they already approximate correct E by summing over matrix M.

Marginal frequencies in wordspace

DSM objects in wordspace (class dsm) include marginal frequencies as well as counts of nonzero cells for rows and columns.

```
> TT$rows
    term       f nnzero
1    cat   22007      5
2    dog   50807      7
3 animal   77053      7
4   time 1156693      7
5 reason   95047      6
6  cause   54739      5
7 effect  133102      6
> TT$cols
...
> TT$globals$N
[1] 199902178
> TT$M # the full co-occurrence matrix
```

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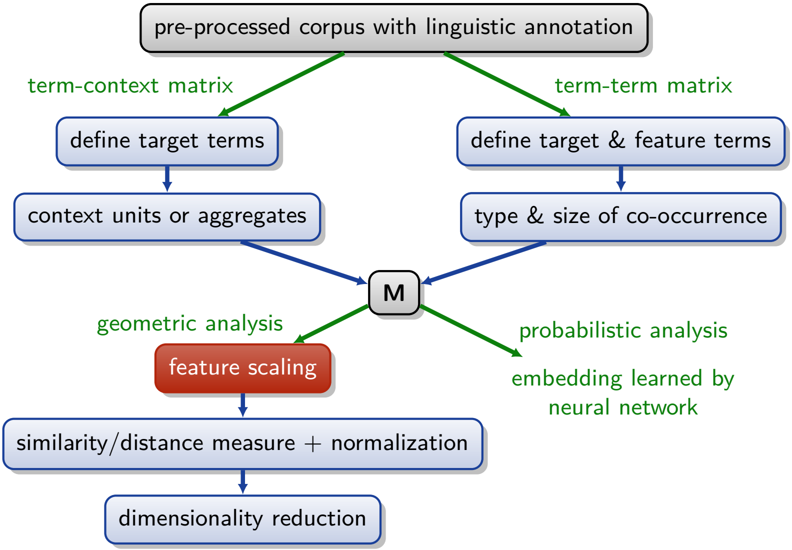

Feature scaling

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- Logarithmic scaling: $O' = \log(O + 1)$
  (cf. Weber-Fechner law for human perception)

Feature scaling

- **M** is often dominated by few very large entries
  (➜ highly skewed frequency distribution due to **Zipf's law**)
- Logarithmic scaling: $O' = \log(O + 1)$
  (cf. Weber-Fechner law for human perception)
- Statistical **association measures** (Evert 2004, 2008) take frequency of target term and feature into account
  - usually based on comparison of observed and expected co-occurrence frequency
  - measures differ in how they balance O and E

Simple association measures

target	feature	O	E
dog	*small*	855	134.34
dog	*domesticated*	29	0.25
dog	*sgjkj*	1	0.00027

Simple association measures

- pointwise Mutual Information (MI)

$$\mathrm{MI} = \log_2 \frac{O}{E}$$

target	feature	O	E	MI
dog	*small*	855	134.34	2.67
dog	*domesticated*	29	0.25	6.85
dog	*sgjkj*	1	0.00027	11.85

Simple association measures

- pointwise Mutual Information (MI)

$$\text{MI} = \log_2 \frac{O}{E}$$

- local MI

$$\text{local-MI} = O \cdot \text{MI} = O \cdot \log_2 \frac{O}{E}$$

target	feature	O	E	MI	local-MI
dog	*small*	855	134.34	2.67	2282.88
dog	*domesticated*	29	0.25	6.85	198.76
dog	*sgjkj*	1	0.00027	11.85	11.85

Simple association measures

- pointwise Mutual Information (MI)

$$\mathrm{MI} = \log_2 \frac{O}{E}$$

- local MI

$$\text{local-MI} = O \cdot \mathrm{MI} = O \cdot \log_2 \frac{O}{E}$$

- t-score

$$t = \frac{O - E}{\sqrt{O}}$$

target	feature	O	E	MI	local-MI	t-score
dog	*small*	855	134.34	2.67	2282.88	24.64
dog	*domesticated*	29	0.25	6.85	198.76	5.34
dog	*sgjkj*	1	0.00027	11.85	11.85	1.00

Other association measures

- simple log-likelihood ($\approx$ local-MI)

$$G^2 = \pm\, 2 \cdot \left(O \cdot \log_2 \frac{O}{E} - (O - E) \right)$$

with positive sign for $O > E$ and negative sign for $O < E$

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- Many other association measures (AMs) available, often based on full contingency tables (see Evert 2008)
  - `http://www.collocations.de/`
  - `http://sigil.r-forge.r-project.org/`

Applying association scores in wordspace

```
> options(digits=3) # print fractional values with limited precision
> dsm.score(TT, score="MI", sparse=FALSE, matrix=TRUE)
        breed   tail   feed   kill important explain  likely
cat      6.21  4.568  3.129  2.801      -Inf  0.0182    -Inf
dog      7.78  3.081  3.922  2.323    -3.774 -1.1888 -0.4958
animal   3.50  2.132  4.747  2.832    -0.674 -0.4677 -0.0966
time    -1.65 -2.236 -0.729 -1.097    -1.728 -1.2382  0.6392
reason  -2.30   -Inf -1.982 -0.388     1.472  4.0368  2.8860
cause    -Inf -0.834   -Inf -2.177     1.900  2.8329  4.0691
effect   -Inf -2.116 -2.468 -2.459     0.791  1.6312  0.9221
```

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effect   -Inf -2.116 -2.468 -2.459     0.791  1.6312  0.9221
```

- ☞ sparseness of matrix representation is lost (try with TC!)
- ☞ cells with score $x = -\infty$ are inconvenient
- ☞ distribution of scores may be even more skewed than co-occurrence frequencies themselves (esp. for G^2)

Sparse association measures

- Sparse association scores are cut off at zero, i.e.

$$f(x) = \begin{cases} x & x > 0 \\ 0 & x \leq 0 \end{cases}$$

- Also known as "positive" scores
  - PPMI = positive pointwise MI (e.g. Bullinaria & Levy 2007)
  - `wordspace` computes sparse AMs by default ➜ `"MI"` = PPMI

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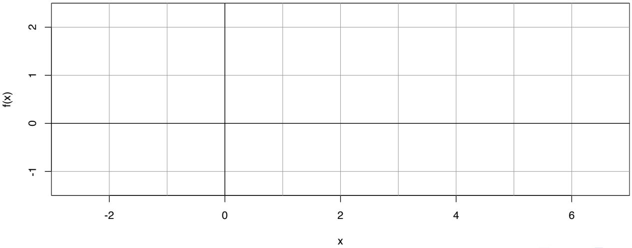
- Further thinning may be beneficial (Polajnar & Clark 2014)
  - apply shifted cutoff threshold $x > \theta$ (Levy *et al.* 2015)
  - keep only k top-scoring features for each target

Score transformations

An additional scale transformation can be applied in order to de-skew association scores:



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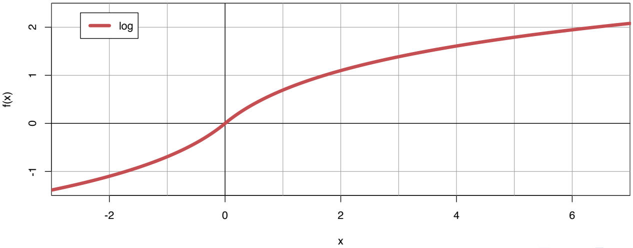
- signed logarithmic transformation

$$f(x) = \pm \log(|x| + 1)$$



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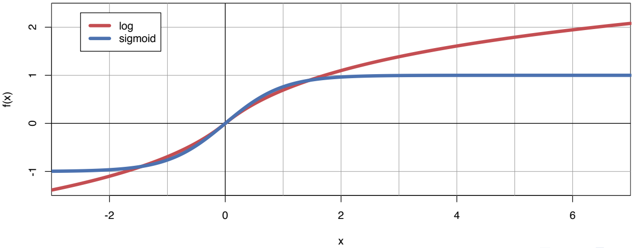
$$f(x) = \tanh x$$



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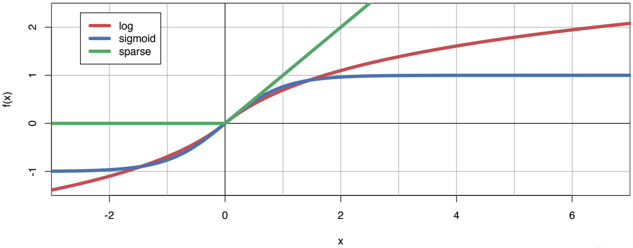

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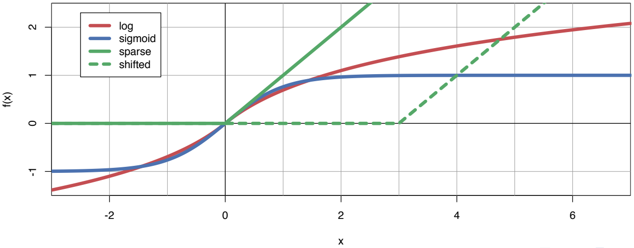
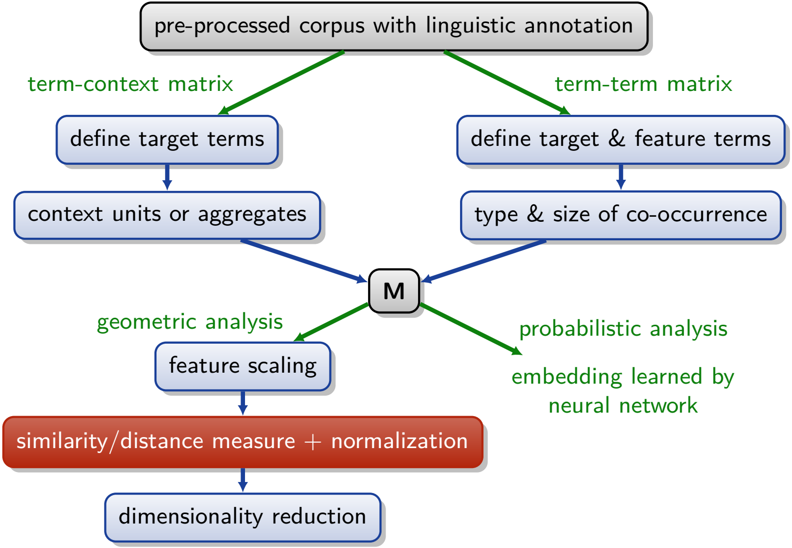


Association scores & transformations in wordspace

```
> dsm.score(TT, score="MI", matrix=TRUE) # PPMI
       breed tail feed kill important explain likely
cat     6.21 4.57 3.13 2.80     0.000  0.0182  0.000
dog     7.78 3.08 3.92 2.32     0.000  0.0000  0.000
animal  3.50 2.13 4.75 2.83     0.000  0.0000  0.000
time    0.00 0.00 0.00 0.00     0.000  0.0000  0.639
reason  0.00 0.00 0.00 0.00     1.472  4.0368  2.886
cause   0.00 0.00 0.00 0.00     1.900  2.8329  4.069
effect  0.00 0.00 0.00 0.00     0.791  1.6312  0.922
> dsm.score(TT, score="simple-ll", matrix=TRUE)
> dsm.score(TT, score="simple-ll", transf="log", matrix=T)
# logarithmic co-occurrence frequency
> dsm.score(TT, score="freq", transform="log", matrix=T)

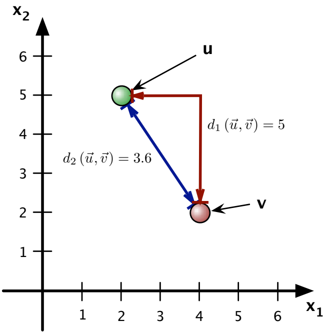
# now try other parameter combinations
> ?dsm.score # read help page for available parameter settings
```

Building a distributional model



Geometric distance = metric

- **Distance** between vectors $\mathbf{u}, \mathbf{v} \in \mathbb{R}^n$ ➜ (dis)similarity
  - $\mathbf{u} = (u_1, \ldots, u_n)$
  - $\mathbf{v} = (v_1, \ldots, v_n)$



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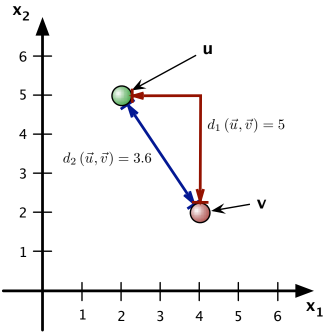
- **Euclidean** distance $d_2(\mathbf{u}, \mathbf{v})$



$$d_2(\mathbf{u}, \mathbf{v}) := \sqrt{(u_1 - v_1)^2 + \dots + (u_n - v_n)^2}$$

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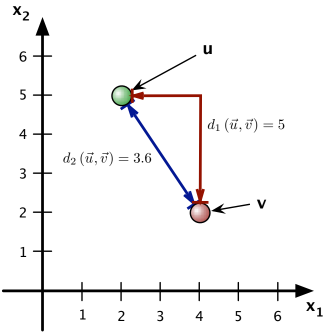
- **Euclidean** distance $d_2(\mathbf{u}, \mathbf{v})$
- "City block" **Manhattan** distance $d_1(\mathbf{u}, \mathbf{v})$



$$d_1(\mathbf{u}, \mathbf{v}) := |u_1 - v_1| + \cdots + |u_n - v_n|$$

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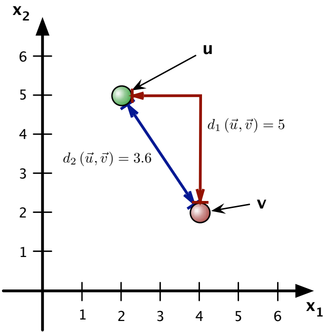
- "City block" **Manhattan** distance $d_1(\mathbf{u}, \mathbf{v})$
- Both are special cases of the **Minkowski** p-distance $d_p(\mathbf{u}, \mathbf{v})$ (for $p \in [1, \infty]$)



$$d_p(\mathbf{u}, \mathbf{v}) := \left(|u_1 - v_1|^p + \cdots + |u_n - v_n|^p\right)^{1/p}$$

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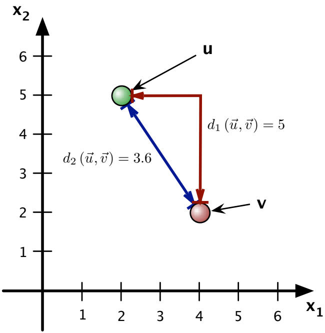

$$d_p(\mathbf{u}, \mathbf{v}) := \left(|u_1 - v_1|^p + \cdots + |u_n - v_n|^p\right)^{1/p}$$

$$d_\infty(\mathbf{u}, \mathbf{v}) = \max\{|u_1 - v_1|, \dots, |u_n - v_n|\}$$

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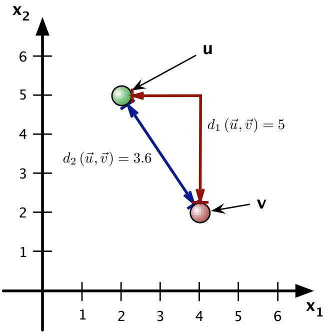
  - $\mathbf{u} = (u_1, \ldots, u_n)$
  - $\mathbf{v} = (v_1, \ldots, v_n)$
- **Hamming** distance $d_0(\mathbf{u}, \mathbf{v})$ not very useful for DSM
- Extension of the Minkowski *p*-**distance** $d_p(\mathbf{u}, \mathbf{v})$ (for $0 \leq p \leq 1$)



$$d_p(\mathbf{u}, \mathbf{v}) := |u_1 - v_1|^p + \cdots + |u_n - v_n|^p$$

$$d_0(\mathbf{u}, \mathbf{v}) = \#\{i \mid u_i \neq v_i\}$$

Computing distances

Preparation: store "scored" matrix in DSM object

```
> TT <- dsm.score(TT, score="freq", transform="log")
```

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Compute distances between individual term pairs . . .

```
> pair.distances(c("cat","cause"), c("animal","effect"),
                 TT, method="euclidean")
  cat/animal cause/effect
        4.16         1.53
```

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 cat/animal cause/effect
       4.16         1.53
```
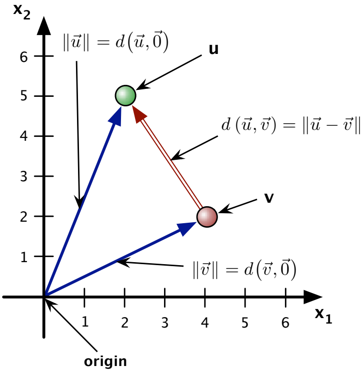

. . . or full distance matrix.

```
> dist.matrix(TT, method="euclidean")
> dist.matrix(TT, method="minkowski", p=4)
```

Distance and vector length = norm

- Intuitively, distance $d(\mathbf{u},\mathbf{v})$ should correspond to length $\|\mathbf{u}-\mathbf{v}\|$ of displacement vector $\mathbf{u}-\mathbf{v}$
  - $d(\mathbf{u},\mathbf{v})$ is a **metric**
  - $\|\mathbf{u}-\mathbf{v}\|$ is a **norm**
  - $\|\mathbf{u}\| = d(\mathbf{u},\mathbf{0})$



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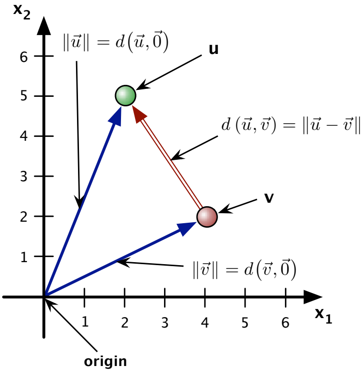
- Any norm-induced metric is **translation-invariant**



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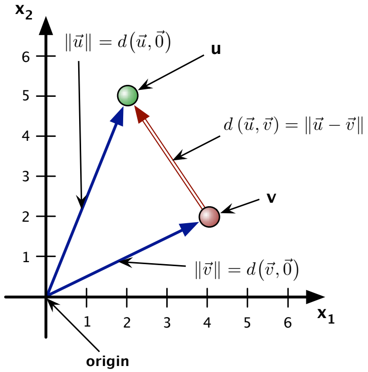
- **Minkowski *p*-norm** with $d_p(\mathbf{u}, \mathbf{v}) = \|\mathbf{u} - \mathbf{v}\|_p$



$$\|\mathbf{u}\|_p := \left(|u_1|^p + \cdots + |u_n|^p\right)^{1/p} \qquad \text{for } 1 \leq p$$

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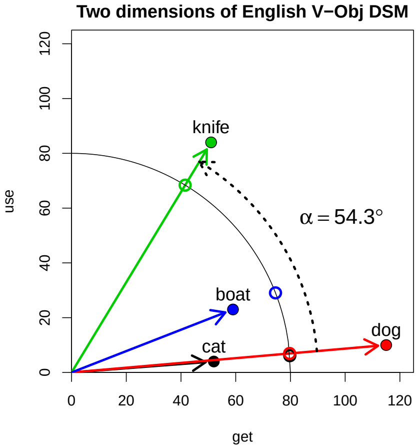
$$\|\mathbf{u}\|_0 = \#\{i \mid u_i \neq 0\} \qquad \|\mathbf{u}\|_\infty = \max\{|u_1|, \ldots, |u_n|\}$$

Normalisation of row vectors

- Part 1: geometric distances only meaningful for vectors of the same length $\|\mathbf{x}\|$



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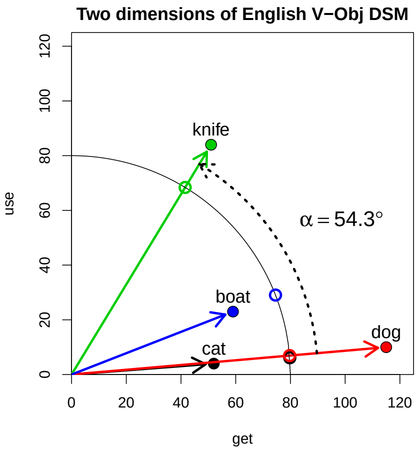
- Normalize by scalar division: $\mathbf{x}' = \mathbf{x}/\|\mathbf{x}\| = (\frac{x_1}{\|\mathbf{x}\|}, \frac{x_2}{\|\mathbf{x}\|}, \dots)$ with $\|\mathbf{x}'\| = 1$
- Norm must be compatible with distance measure!



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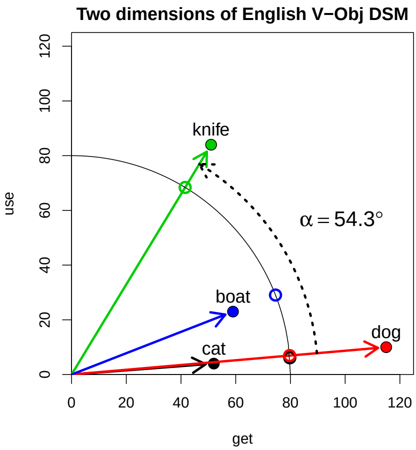
- Norm must be compatible with distance measure!
- Special case: scale $\mathbf{x} \geq 0$ to stochastic vector with
  $$\|\mathbf{x}\|_1 = |x_1| + \dots + |x_n|$$
  ➜ probabilistic interpretation



Norms and normalization

```
> rowNorms(TT$S, method="euclidean")
   cat    dog animal   time reason  cause effect
  6.90   8.96   8.82  10.29   8.13   6.86   6.52
```

```
> TT <- dsm.score(TT, score="freq", transform="log",
                  normalize=TRUE, method="euclidean")
> rowNorms(TT$S, method="euclidean") # all = 1 now
> dist.matrix(TT, method="euclidean")
         cat   dog animal  time reason cause effect
cat    0.000 0.224  0.473 0.782  1.121 1.239  1.161
dog    0.224 0.000  0.398 0.698  1.065 1.179  1.113
animal 0.473 0.398  0.000 0.426  0.841 0.971  0.860
time   0.782 0.698  0.426 0.000  0.475 0.585  0.502
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effect 1.161 1.113  0.860 0.502  0.198 0.224  0.000
```

Distance measures for non-negative vectors

- Information theory: **Kullback-Leibler** (KL) **divergence** for stochastic vectors (non-negative $\mathbf{x} \geq 0$ and $\|\mathbf{x}\|_1 = 1$)

$$D(\mathbf{u}\|\mathbf{v}) = \sum_{i=1}^{n} u_i \cdot \log_2 \frac{u_i}{v_i}$$

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- Properties of KL divergence
  - most appropriate for a probabilistic interpretation of $\mathbf{M}$
  - zeroes in $\mathbf{v}$ without corresponding zeroes in $\mathbf{u}$ are problematic
  - **not symmetric**, unlike geometric distance measures
  - alternatives: skew divergence, Jensen-Shannon divergence

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  - **not symmetric**, unlike geometric distance measures
  - alternatives: skew divergence, Jensen-Shannon divergence

- A symmetric distance metric (Endres & Schindelin 2003)

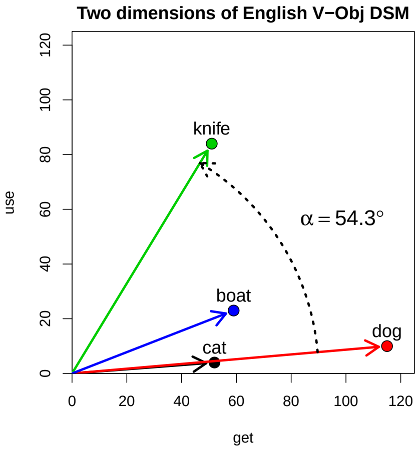
$$D_{\mathbf{uv}} = D(\mathbf{u}\|\mathbf{z}) + D(\mathbf{v}\|\mathbf{z}) \quad \text{with} \quad \mathbf{z} = \frac{\mathbf{u}+\mathbf{v}}{2}$$

Similarity measures

- Angle α between vectors $\mathbf{u}, \mathbf{v} \in \mathbb{R}^n$ is given by

$$\cos \alpha = \frac{\sum_{i=1}^{n} u_i \cdot v_i}{\sqrt{\sum_i u_i^2} \cdot \sqrt{\sum_i v_i^2}}$$

$$= \frac{\mathbf{u}^T \mathbf{v}}{\|\mathbf{u}\|_2 \cdot \|\mathbf{v}\|_2}$$

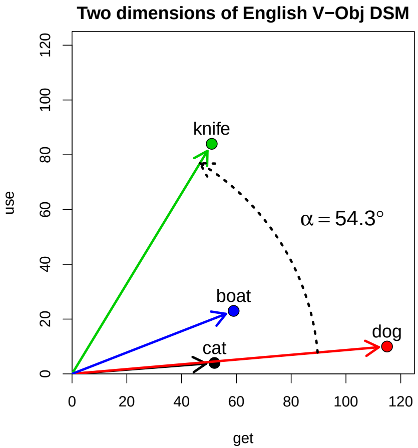


Similarity measures

- Angle α between vectors $\mathbf{u}, \mathbf{v} \in \mathbb{R}^n$ is given by

$$\cos\alpha = \frac{\sum_{i=1}^{n} u_i \cdot v_i}{\sqrt{\sum_i u_i^2} \cdot \sqrt{\sum_i v_i^2}} = \frac{\mathbf{u}^T \mathbf{v}}{\|\mathbf{u}\|_2 \cdot \|\mathbf{v}\|_2}$$

- **cosine** measure of similarity: $\cos\alpha$
  - $\cos\alpha = 1$ ➜ collinear
  - $\cos\alpha = 0$ ➜ orthogonal
- Corresponding metric: **angular distance** α



Euclidean distance or cosine similarity?

$$
\begin{aligned}
d_2(\mathbf{u}, \mathbf{v}) = \|\mathbf{u} - \mathbf{v}\|_2 &= \sqrt{\sum_i (u_i - v_i)^2} \\
&= \sqrt{\sum_i u_i^2 + \sum_i v_i^2 - 2 \sum_i u_i v_i} \\
&= \sqrt{\|\mathbf{u}\|_2^2 + \|\mathbf{v}\|_2^2 - 2\,\mathbf{u}^T \mathbf{v}} \\
&= \sqrt{2 - 2\cos\phi}
\end{aligned}
$$

☞ $d_2(\mathbf{u}, \mathbf{v})$ is a monotonically increasing function of ϕ

Similarity measures for non-negative vectors

- Generalized **Jaccard coefficient** = shared features

$$J(\mathbf{u}, \mathbf{v}) = \frac{\sum_{i=1}^{n} \min\{u_i, v_i\}}{\sum_{i=1}^{n} \max\{u_i, v_i\}}$$

- $1 - J(\mathbf{u}, \mathbf{v})$ is a distance **metric** (Kosub 2016)

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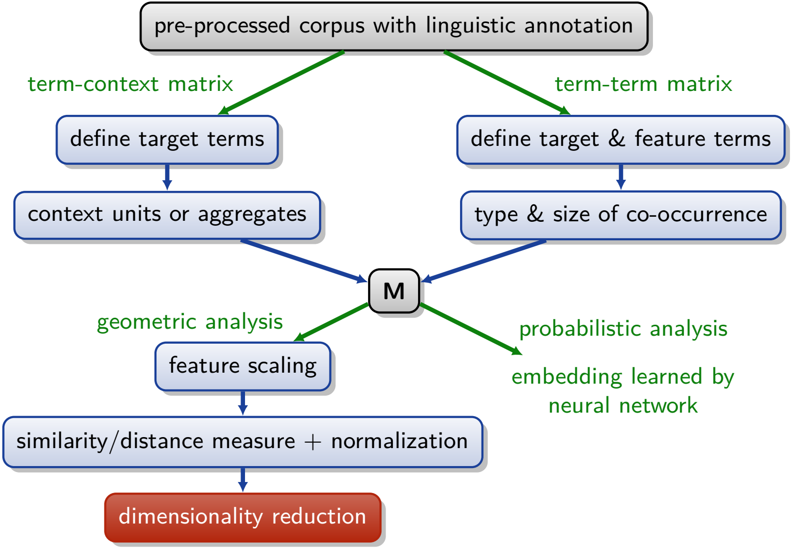
- $1 - J(\mathbf{u},\mathbf{v})$ is a distance **metric** (Kosub 2016)

- An asymmetric measure of feature **overlap** (Clarke 2009)

$$o(\mathbf{u},\mathbf{v}) = \frac{\sum_{i=1}^{n} \min\{u_i, v_i\}}{\sum_{i=1}^{n} u_i}$$

Building a distributional model



Dimensionality reduction = model compression

- Co-occurrence matrix **M** is often unmanageably large and can be extremely sparse
  - Google Web1T5: 1M × 1M matrix with one trillion cells, of which less than 0.05% contain nonzero counts (Evert 2010)

➥ Compress matrix by reducing dimensionality (= rows)

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  - measured by entropy, chi-squared test, nonzero count, ...
  - may select similar dimensions and discard valuable information

- **Projection** into (linear) subspace
  - principal component analysis (PCA)
  - independent component analysis (ICA)
  - random indexing (RI)
  - ☞ intuition: preserve distances between data points

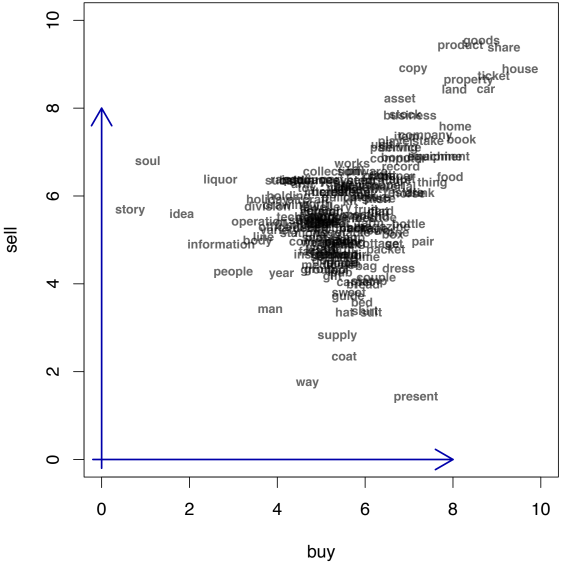
Dimensionality reduction & latent dimensions

Landauer & Dumais (1997) claim that LSA dimensionality reduction (and related PCA technique) uncovers **latent dimensions** by exploiting correlations between features.

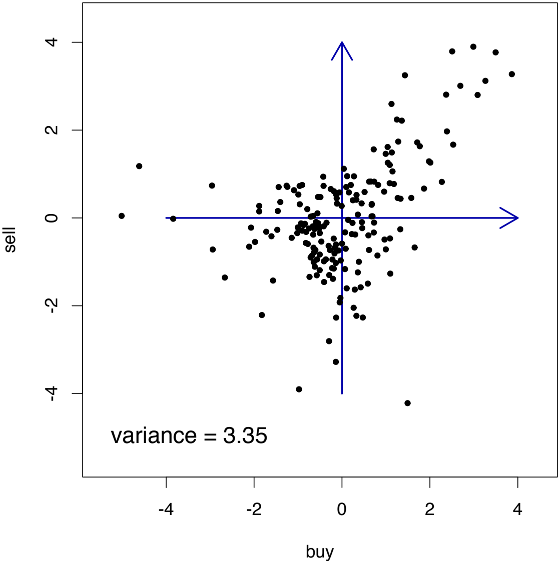
- Example: term-term matrix
- V-Obj co-oc. extracted from BNC
  - targets = noun lemmas
  - features = verb lemmas
- feature scaling: association scores (SketchEngine log Dice)
- $k = 186$ nouns with $f_{\text{buy}} + f_{\text{sell}} \geq 25$
- $n = 2$ dimensions: *buy* and *sell*

noun	*buy*	*sell*
antique	5.12	5.50
bread	5.96	3.99
computer	6.75	6.83
factory	4.95	4.72
group	4.93	4.28
jewellery	5.11	5.73
mill	5.14	5.41
people	3.00	4.26
record	6.81	6.68
souvenir	5.45	4.67
ticket	8.93	8.74

Dimensionality reduction & latent dimensions



Dimensionality reduction by PCA



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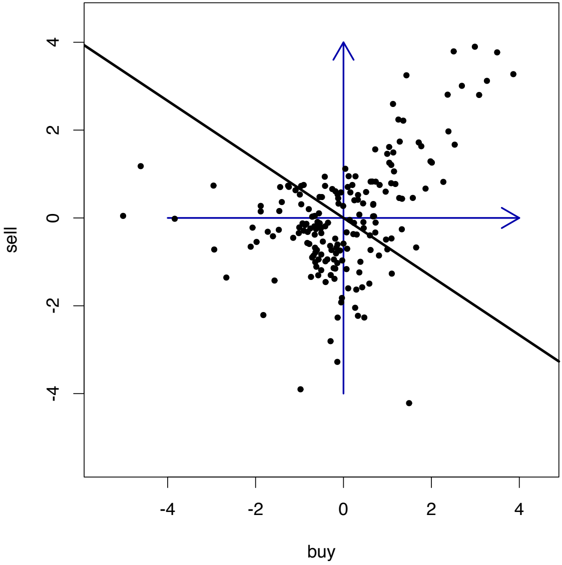

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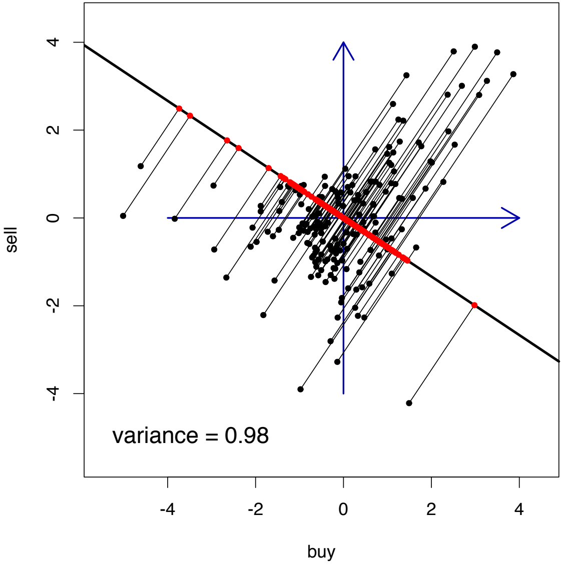

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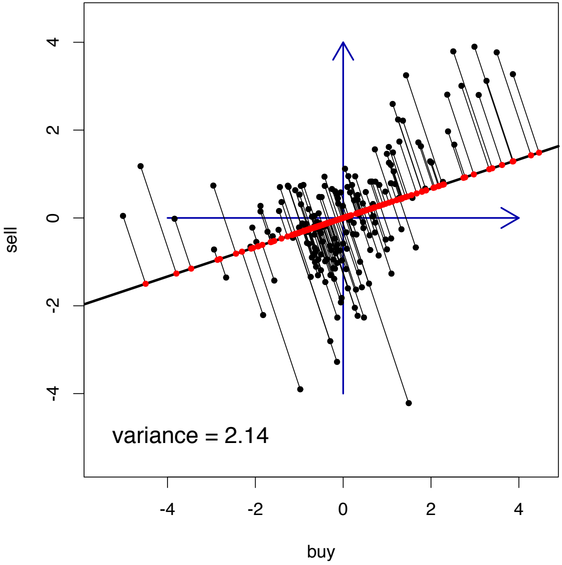

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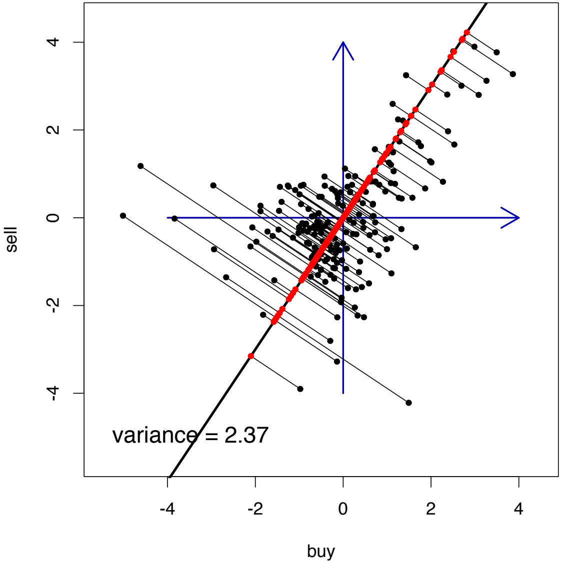

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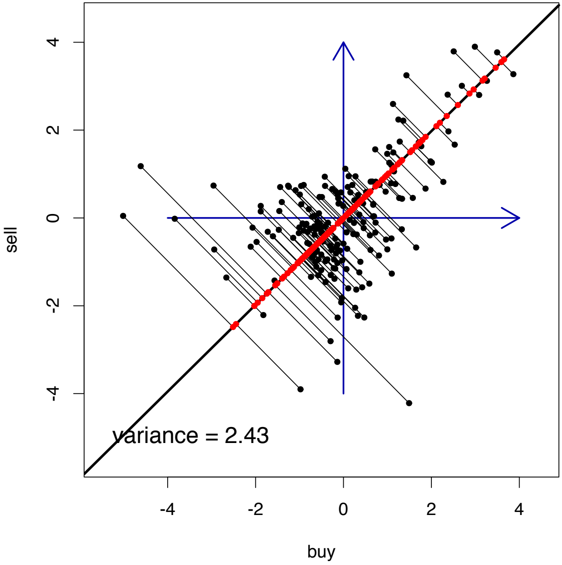


Dimensionality reduction in practice

```
# SVD is the algorithm behind PCA dimensionality reduction
> TT2 <- dsm.projection(TT, n=2, method="svd")
> TT2
         svd1    svd2
cat    -0.733 -0.6615
dog    -0.782 -0.6110
animal -0.914 -0.3606
time   -0.993  0.0302
reason -0.889  0.4339
cause  -0.817  0.5615
effect -0.871  0.4794

> x <- TT2[, 1] # first latent dimension
> y <- TT2[, 2] # second latent dimension
> plot(x, y, pch=20, col="red",
       xlim=extendrange(x), ylim=extendrange(y))
> text(x, y, rownames(TT2), pos=3)
```

Dimensionality reduction as matrix factorization

- PCA is based on **singular value decomposition** (**SVD**), which factorises any matrix **M** into

$$\mathbf{M} = \mathbf{U}\boldsymbol{\Sigma}\mathbf{V}^T$$

where **U** and **V** are orthogonal and $\boldsymbol{\Sigma}$ is a diagonal matrix of **singular values** $\sigma_1 \geq \sigma_2 \geq \cdots \geq \sigma_m > 0$

$$\begin{array}{cc} & n \\ k & \begin{bmatrix} & & \\ & \mathbf{M} & \\ & & \end{bmatrix} \end{array} = \begin{array}{cc} & m \\ k & \begin{bmatrix} & & \\ & \mathbf{U} & \\ & & \end{bmatrix} \end{array} \cdot \begin{array}{cc} & m \\ m & \begin{bmatrix} \sigma_1 & & \\ & \ddots & \\ & \boldsymbol{\Sigma} & \sigma_m \end{bmatrix} \end{array} \cdot \begin{array}{cc} & n \\ m & \begin{bmatrix} & & \\ & \mathbf{V}^T & \\ & & \end{bmatrix} \end{array}$$

Dimensionality reduction as matrix factorization

- Columns $\mathbf{a}_i$ of $\mathbf{U}$ and $\mathbf{b}_i$ of $\mathbf{V}$ (**singular vectors**) are orthogonal ($\mathbf{a}_i^T \mathbf{a}_j = 0$) and of unit length ($\|\mathbf{a}_i\| = 1$)
- Key property: **truncated SVD** gives best least-squares approximation in r-dimensional subspace

$$\mathbf{U}_r \mathbf{\Sigma}_r \mathbf{V}_r^T = \begin{bmatrix} \vdots & & \vdots \\ \vdots & & \vdots \\ \mathbf{a}_1 & \cdots & \mathbf{a}_r \\ \vdots & & \vdots \\ \vdots & \mathbf{U}_r & \vdots \end{bmatrix} \cdot \begin{bmatrix} \sigma_1 & & \\ & \ddots & \\ & \mathbf{\Sigma}_r & \sigma_r \end{bmatrix} \cdot \begin{bmatrix} \cdots & \cdots & \mathbf{b}_1 & \cdots & \cdots \\ \mathbf{V}_r^T & & \vdots & & \\ \cdots & \cdots & \mathbf{b}_r & \cdots & \cdots \end{bmatrix}$$

Dimensionality reduction as matrix factorization

- Truncated SVD as orthogonal projection

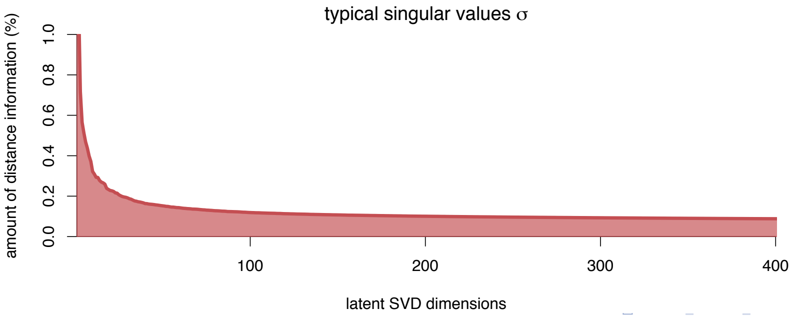
$$\mathbf{M}\mathbf{V}_r = \mathbf{U}_r\mathbf{\Sigma}_r = \begin{bmatrix} \vdots & & \vdots \\ \sigma_1\mathbf{a}_1 & \cdots & \sigma_r\mathbf{a}_r \\ \vdots & & \vdots \end{bmatrix}$$

  ➜ `method="svd"` in `dsm.projection()`

- $\sigma_1^2 \geq \sigma_2^2 \geq \ldots =$ amount of distance information (i.e. variance of $\mathbf{M}$) captured by each **latent dimension**

Scaling latent dimensions

- Truncated SVD omits latent dimensions that capture relatively little distance information (here $r = 400$)



Scaling latent dimensions

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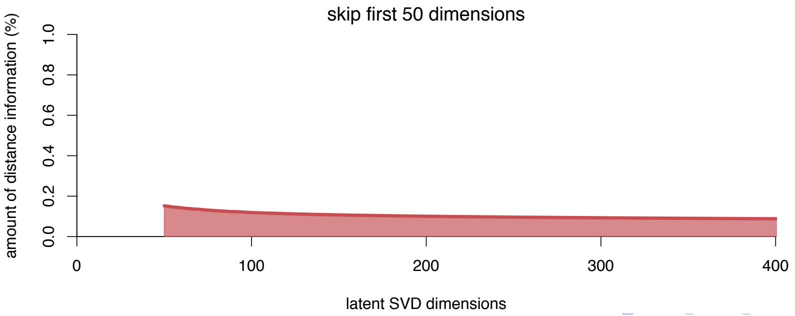

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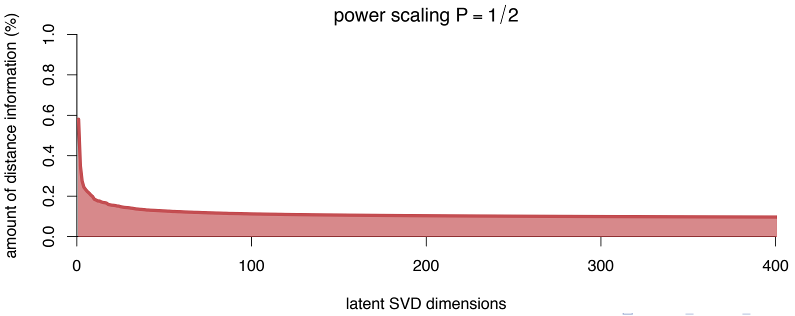
  - Bullinaria & Levy (2012) report positive effect



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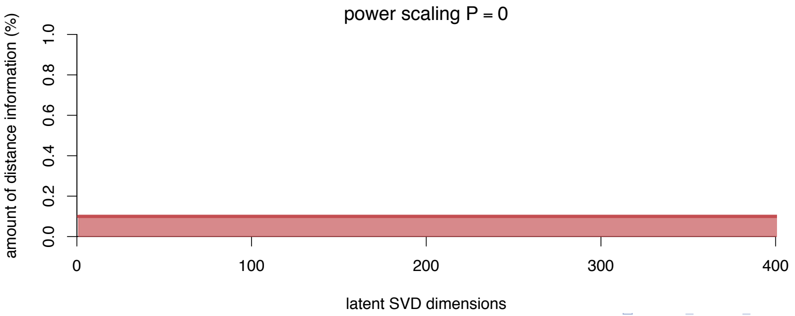
  - esp. with $P = 0$ to equalize dimensions (**whitening**)



Power-scaling in practice

```
> TT2 <- dsm.projection(TT, n=2, method="svd", power=0)
> TT2
         svd1    svd2
cat    -0.322 -0.5110
dog    -0.343 -0.4721
animal -0.401 -0.2786
time   -0.436  0.0233
reason -0.390  0.3353
cause  -0.359  0.4338
effect -0.383  0.3704

# power-scaling can also be applied post-hoc
> sigma <- attr(TT2, "sigma")         # singular values
> scaleMargins(TT2, cols=sigma^0.5) # P = 1/2
> scaleMargins(TT2, cols=sigma)      # unscaled (P = 1)
```

Other matrix factorization techniques

- **Non-negative matrix factorization** (**NMF**)
  - **U** and **V** are stochastic matrices ($\mathbf{a}_i \geq 0$ and $\|\mathbf{a}_i\|_1 = 1$)
  - cross-entropy instead of least-squares approximation
  - iterative algorithm with random initialisation for rank-r approximation ($\neq$ sequence of ordered components)

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- NMF of term-document matrix $\iff$ LDA **topic model**

$$\mathbf{U}\mathbf{\Sigma}\mathbf{V}^T = \sigma_1 \mathbf{a}_1 \mathbf{b}_1^T + \sigma_2 \mathbf{a}_2 \mathbf{b}_2^T + \sigma_3 \mathbf{a}_3 \mathbf{b}_3^T + \dots$$

  - $\mathbf{a}_i$ = probability distribution of words in i-th topic
  - $\mathbf{b}_i$ = distribution of topic across documents

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$$\mathbf{U\Sigma V}^T = \sigma_1 \mathbf{a}_1 \mathbf{b}_1^T + \sigma_2 \mathbf{a}_2 \mathbf{b}_2^T + \sigma_3 \mathbf{a}_3 \mathbf{b}_3^T + \dots$$

  - $\mathbf{a}_i$ = probability distribution of words in i-th topic
  - $\mathbf{b}_i$ = distribution of topic across documents
- Levy *et al.* (2015, 213) show that **word2vec** embeddings implicitly factorize a shifted PPMI matrix
  - sigmoid loss function, weighted towards high frequencies
  - similarly, **GloVe** (Pennington *et al.* 2014) factorizes matrix of conditional probabilities with a frequency-weighted least-squares approximation

Outline

Scaling up to the real world

- So far, we have worked on minuscule **toy models**
- ☞ We want to scale up to **real world** data sets now

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- Example 1: window-based DSM on BNC content words
  - 83,926 lemma types with $f \geq 10$
  - term-term matrix with $83{,}926 \cdot 83{,}926 = 7$ billion entries
  - standard representation requires 56 GB of RAM (8-byte floats)
  - only 22.1 million non-zero entries ($= 0.32\%$)

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- Example 2: Google Web 1T 5-grams (1 trillion words)
  - more than 1 million word types with $f \geq 2500$
  - term-term matrix with 1 trillion entries requires 8 TB RAM
  - only 400 million non-zero entries ($= 0.04\%$)

Sparse matrix representation

- Invented example of a **sparsely populated** DSM matrix

	eat	get	hear	kill	see	use
boat	·	59	·	·	39	23
cat	·	·	·	26	58	·
cup	·	98	·	·	·	·
dog	33	·	42	·	83	·
knife	·	·	·	·	·	84
pig	9	·	·	27	·	·

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boat	·	59	·	·	39	23
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knife	·	·	·	·	·	84
pig	9	·	·	27	·	·

- Store only non-zero entries in compact **sparse matrix format**

row	col	value
1	2	59
1	5	39
1	6	23
2	4	26
2	5	58
3	2	98

row	col	value
4	1	33
4	3	42
4	5	83
5	6	84
6	1	9
6	4	27

Working with sparse matrices

- Compressed format: each row index (or column index) stored only once, followed by non-zero entries in this row (or column)
  - convention: **column-major** matrix (data stored by columns)

- Specialised algorithms for sparse matrix algebra
  - especially matrix multiplication, solving linear systems, etc.
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  - especially matrix multiplication, solving linear systems, etc.
  - take care to avoid operations that create a dense matrix!
- **R** implementation: `Matrix` package
  - essential for real-life distributional semantics
  - `wordspace` provides additional support for sparse matrices (vector distances, sparse SVD, …)
- Other software: Matlab, Octave, Python + SciPy

Outline

Triplet tables

- A sparse DSM matrix can be represented as a table of triplets (target, feature, co-occurrence frequency)
  - for syntactic co-occurrence and term-document matrices, marginals can be computed from a complete triplet table
  - for surface and textual co-occurrence, marginals have to be provided in separate files (see `?read.dsm.triplet`)

noun	rel	verb	f	mode
dog	subj	bite	3	spoken
dog	subj	bite	12	written
dog	obj	bite	4	written
dog	obj	stroke	3	written
...	...	...	...	...

- `DSM_VerbNounTriples_BNC` contains additional information
  - syntactic relation between noun and verb
  - written or spoken part of the British National Corpus

Constructing a DSM from a triplet table

- Additional information can be used for filtering (verb-object relation), or aggregate frequencies (spoken + written BNC)

```
> tri <- subset(DSM_VerbNounTriples_BNC, rel == "obj")
```

- Construct DSM object from triplet input
  - `raw.freq=TRUE` indicates raw co-occurrence frequencies (rather than a pre-weighted DSM)
  - constructor aggregates counts from duplicate entries
  - marginal frequencies are automatically computed

```
> VObj <- dsm(target=tri$noun, feature=tri$verb,
              score=tri$f, raw.freq=TRUE)
> VObj  # inspect marginal frequencies (e.g. head(VObj$rows, 20))
```

Exploring the DSM

```
> VObj <- dsm.score(VObj, score="MI", normalize=TRUE)

> nearest.neighbours(VObj, "dog") # angular distance
   horse       cat    animal    rabbit      fish       guy
    73.9      75.9      76.2      77.0      77.2      78.5
 cichlid       kid       bee  creature
    78.6      79.0      79.1      79.5

> nearest.neighbours(VObj, "dog", method="manhattan")
# NB: we used an incompatible Euclidean normalization!

> VObj50 <- dsm.projection(VObj, n=50, method="svd")
> nearest.neighbours(VObj50, "dog")
```

Practice

- Code examples and further explanations: `hands_on_day2.R`
- How many different models can you build from `DSM_VerbNounTriples_BNC`?
  - apply different filters, scores, transformations and metrics
  - ☞ explore nearest neighbours of selected words
- Build real-life DSMs from pre-compiled co-occurrence data
  - http://wordspace.collocations.de/doku.php/course:material
  - load pre-compiled matrix and apply different parameters
  - ☞ compare nearest neighbours or semantic maps
- Learn how to import your own co-occurrence data
  ☞ `hands_on_day2_input_formats.R`
  - download example data sets to subdirectory `data/`
- Explore matrix factorization techniques
  ☞ `hands_on_day2_matrix_factorization.R`

Outline

Some well-known DSM examples

Latent Semantic Analysis (Landauer & Dumais 1997)

- term-context matrix with document context
- weighting: log term frequency and term entropy
- distance measure: cosine
- dimensionality reduction: SVD

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- distance measure: cosine
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Hyperspace Analogue to Language (Lund & Burgess 1996)

- term-term matrix with surface context
- structured (left/right) and distance-weighted frequency counts
- distance measure: Minkowski metric ($1 \leq p \leq 2$)
- dimensionality reduction: feature selection (high variance)

Some well-known DSM examples

Infomap NLP (Widdows 2004)

- term-term matrix with unstructured surface context
- weighting: none
- distance measure: cosine
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- term-term matrix with unstructured surface context
- weighting: none
- distance measure: cosine
- dimensionality reduction: SVD

Random Indexing (Karlgren & Sahlgren 2001)

- term-term matrix with unstructured surface context
- weighting: various methods
- distance measure: various methods
- dimensionality reduction: random indexing (RI)

Some well-known DSM examples

Dependency Vectors (Padó & Lapata 2007)

- term-term matrix with unstructured dependency context
- weighting: log-likelihood ratio
- distance measure: PPMI-weighted Dice (Lin 1998)
- dimensionality reduction: none

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- term-term matrix with unstructured dependency context
- weighting: log-likelihood ratio
- distance measure: PPMI-weighted Dice (Lin 1998)
- dimensionality reduction: none

Distributional Memory (Baroni & Lenci 2010)

- term-term matrix with structured and unstructered dependencies + knowledge patterns
- weighting: local-MI on type frequencies of link patterns
- distance measure: cosine
- dimensionality reduction: none

. . . and an unexpected application

Authorship attribution (Burrows 2002)

- Burrows's Delta method is very popular in modern literary stylometry and authorship attribution (Evert *et al.* 2017)
- document-term matrix with word forms as features
- weighting: relative frequency of word form in document
- feature selection: 200–5,000 most frequent words (mfw)
- columns are standardized ($\mu = 0$, $\sigma^2 = 1$) ➜ z-scores
- clustering of documents based on various distance metrics (or nearest-neighbour classifier for known authors)
- dimensionality reduction: none
- main result: angle/cosine $\succ$ Manhattan $\succ$ Euclidean

Outline

Latent Semantic Analysis (Landauer & Dumais 1997)

- Corpus: 30,473 articles from Grolier's *Academic American Encyclopedia* (4.6 million words in total)
  - ☞ articles were limited to first 2,000 characters
- Word-article frequency matrix for 60,768 words
  - row vector shows frequency of word in each article
- Logarithmic frequencies scaled by word entropy
- Reduced to 300 dim. by singular value decomposition (SVD)
  - borrowed from LSI (Dumais *et al.* 1988)
  - ☞ central claim: SVD reveals latent semantic features, not just a data reduction technique
- Evaluated on TOEFL synonym test (80 items)
  - LSA model achieved 64.4% correct answers
  - also simulation of learning rate based on TOEFL results

Word Space (Schütze 1992, 1993, 1998)

- Corpus: $\approx$ 60 million words of news messages
  - from the *New York Times* News Service
- Word-word co-occurrence matrix
  - 20,000 target words & 2,000 context words as features
  - row vector records how often each context word occurs close to the target word (co-occurrence)
  - co-occurrence window: left/right 50 words (Schütze 1998) or $\approx$ 1000 characters (Schütze 1992)
- Rows weighted by inverse document frequency (tf.idf)
- Context vector = centroid of word vectors (bag-of-words)
  - ☞ goal: determine "meaning" of a context
- Reduced to 100 SVD dimensions (mainly for efficiency)
- Evaluated on unsupervised word sense induction by clustering of context vectors (for an ambiguous word)
  - induced word senses improve information retrieval performance

HAL (Lund & Burgess 1996)

- HAL = Hyperspace Analogue to Language
- Corpus: 160 million words from newsgroup postings
- Word-word co-occurrence matrix
  - same 70,000 words used as targets and features
  - co-occurrence window of 1 – 10 words
- Separate counts for left and right co-occurrence
  - i.e. the context is *structured*
- In later work, co-occurrences are weighted by (inverse) distance (Li *et al.* 2000)
  - but no dimensionality reduction
- Applications include construction of semantic vocabulary maps by multidimensional scaling to 2 dimensions

HAL (Lund & Burgess 1996)

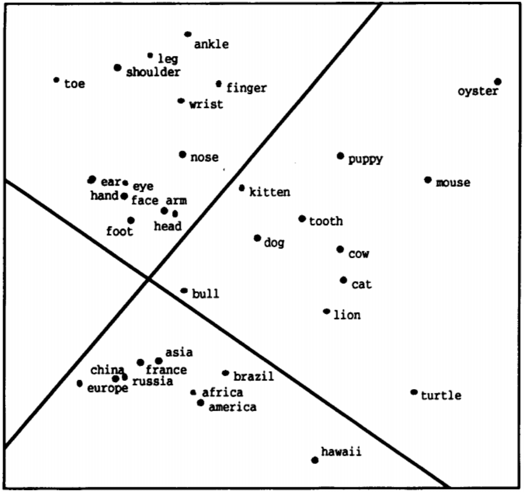

Figure 2. Multidimensional scalirg of co-occurrence vectors.

References I

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