Distributional Semantic Models
Part 2: The parameters of a DSM

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Outline

DSM parameters
- A taxonomy of DSM parameters
- Examples

Building a DSM
- Sparse matrices
- Example: a verb-object DSM
A **distributional semantic model** (DSM) is a scaled and/or transformed co-occurrence matrix $\mathbf{M}$, such that each row $\mathbf{x}$ represents the distribution of a target term across contexts.

<table>
<thead>
<tr>
<th>Term</th>
<th>get</th>
<th>see</th>
<th>use</th>
<th>hear</th>
<th>eat</th>
<th>kill</th>
</tr>
</thead>
<tbody>
<tr>
<td>knife</td>
<td>0.027</td>
<td>-0.024</td>
<td>0.206</td>
<td>-0.022</td>
<td>-0.044</td>
<td>-0.042</td>
</tr>
<tr>
<td>cat</td>
<td>0.031</td>
<td>0.143</td>
<td>-0.243</td>
<td>-0.015</td>
<td>-0.009</td>
<td>0.131</td>
</tr>
<tr>
<td>dog</td>
<td>-0.026</td>
<td>0.021</td>
<td>-0.212</td>
<td>0.064</td>
<td>0.013</td>
<td>0.014</td>
</tr>
<tr>
<td>boat</td>
<td>-0.022</td>
<td>0.009</td>
<td>-0.044</td>
<td>-0.040</td>
<td>-0.074</td>
<td>-0.042</td>
</tr>
<tr>
<td>cup</td>
<td>-0.014</td>
<td>-0.173</td>
<td>-0.249</td>
<td>-0.099</td>
<td>-0.119</td>
<td>-0.042</td>
</tr>
<tr>
<td>pig</td>
<td>-0.069</td>
<td>0.094</td>
<td>-0.158</td>
<td>0.000</td>
<td>0.094</td>
<td>0.265</td>
</tr>
<tr>
<td>banana</td>
<td>0.047</td>
<td>-0.139</td>
<td>-0.104</td>
<td>-0.022</td>
<td>0.267</td>
<td>-0.042</td>
</tr>
</tbody>
</table>

**Term** = word, lemma, phrase, morpheme, word pair, …
General definition of DSMs

Mathematical notation:

- $k \times n$ co-occurrence matrix $\mathbf{M} \in \mathbb{R}^{k \times n}$ (example: $7 \times 6$)
  - $k$ rows = target terms
  - $n$ columns = features or dimensions

\[
\mathbf{M} = \begin{bmatrix}
m_{11} & m_{12} & \cdots & m_{1n} \\
m_{21} & m_{22} & \cdots & m_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
m_{k1} & m_{k2} & \cdots & m_{kn}
\end{bmatrix}
\]

- distribution vector $\mathbf{m}_i = i$-th row of $\mathbf{M}$, e.g. $\mathbf{m}_3 = \mathbf{m}_{\text{dog}} \in \mathbb{R}^n$
- components $\mathbf{m}_i = (m_{i1}, m_{i2}, \ldots, m_{in}) = \text{features of } i$-th term:

\[
\mathbf{m}_3 = (-0.026, 0.021, -0.212, 0.064, 0.013, 0.014)
\]

\[
= (m_{31}, m_{32}, m_{33}, m_{34}, m_{35}, m_{36})
\]
Outline

DSM parameters
   A taxonomy of DSM parameters
   Examples

Building a DSM
   Sparse matrices
   Example: a verb-object DSM
Overview of DSM parameters

- Pre-processed corpus with linguistic annotation
  - Define target terms
    - Context tokens or types
  - Define target & feature terms
    - Type & size of co-occurrence
  - Term-term matrix
  - Term-context matrix

- Geometric analysis
  - Feature scaling
    - Similarity/distance measure + normalization
      - Dimensionality reduction

- Probabilistic analysis
  - Embedding learned by neural network
Term-context matrix

Term-context matrix records frequency of term in each individual context (e.g. sentence, document, Web page, encyclopaedia article)

\[ F = \begin{bmatrix} \cdots & f_1 & \cdots \\ \cdots & f_2 & \cdots \\ \vdots & \vdots & \vdots \\ \cdots & f_k & \cdots \end{bmatrix} \]

\[
\begin{array}{c|c|c|c|c|c|c|c}
 & Felidae & Pet & Feral & Bloat & Philosophy & Kant & Back pain \\
\hline
\text{cat} & 10 & 10 & 7 & - & - & - & - \\
\text{dog} & - & 10 & 4 & 11 & - & - & - \\
\text{animal} & 2 & 15 & 10 & 2 & - & - & - \\
\text{time} & 1 & - & - & - & 2 & 1 & - \\
\text{reason} & - & 1 & - & - & 1 & 4 & 1 \\
\text{cause} & - & - & - & 2 & 1 & 2 & 6 \\
\text{effect} & - & - & - & 1 & - & 1 & - \\
\end{array}
\]

> TC <- DSM_TermContext
> head(TC, Inf)  # extract full co-oc matrix from DSM object
Term-term matrix

**Term-term matrix** records co-occurrence frequencies with feature terms for each target term

\[
M = \begin{bmatrix}
\cdots & m_1 & \cdots \\
\cdots & m_2 & \cdots \\
\vdots & \vdots & \ddots \\
\cdots & m_k & \cdots 
\end{bmatrix}
\]

<table>
<thead>
<tr>
<th></th>
<th>breed</th>
<th>tail</th>
<th>feed</th>
<th>kill</th>
<th>important</th>
<th>explain</th>
<th>likely</th>
</tr>
</thead>
<tbody>
<tr>
<td>cat</td>
<td>83</td>
<td>17</td>
<td>7</td>
<td>37</td>
<td>–</td>
<td>1</td>
<td>–</td>
</tr>
<tr>
<td>dog</td>
<td>561</td>
<td>13</td>
<td>30</td>
<td>60</td>
<td>1</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>animal</td>
<td>42</td>
<td>10</td>
<td>109</td>
<td>134</td>
<td>13</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>time</td>
<td>19</td>
<td>9</td>
<td>29</td>
<td>117</td>
<td>81</td>
<td>34</td>
<td>109</td>
</tr>
<tr>
<td>reason</td>
<td>1</td>
<td>–</td>
<td>2</td>
<td>14</td>
<td>68</td>
<td>140</td>
<td>47</td>
</tr>
<tr>
<td>cause</td>
<td>–</td>
<td>1</td>
<td>–</td>
<td>4</td>
<td>55</td>
<td>34</td>
<td>55</td>
</tr>
<tr>
<td>effect</td>
<td>–</td>
<td>–</td>
<td>1</td>
<td>6</td>
<td>60</td>
<td>35</td>
<td>17</td>
</tr>
</tbody>
</table>

> TT <- DSM_TermTerm
> head(TT, Inf)
Term-term matrix

Some footnotes:

- Often target terms \( \neq \) feature terms
  - e.g. nouns described by co-occurrences with verbs as features
  - identical sets of target & feature terms \( \rightarrow \) symmetric matrix

- Different types of co-occurrence (Evert 2008)
  - surface context (word or character window)
  - textual context (non-overlapping segments)
  - syntactic context (dependency relation)

- Can be seen as smoothing of term-context matrix
  - average over similar contexts (with same context terms)
  - data sparseness reduced, except for small windows
  - we will take a closer look at the relation between term-context and term-term models in part 5 of this tutorial
Overview of DSM parameters

- Define target terms
  - Context tokens or types
  - Type & size of co-occurrence

- Define target & feature terms
  - Term-context matrix
  - Term-term matrix

- Feature scaling
  - Similarity/distance measure + normalization
  - Dimensionality reduction

- Geometric analysis

- Probabilistic analysis
  - Embedding learned by neural network
Definition of target and feature terms

- Choice of linguistic unit
  - words
  - bigrams, trigrams, ...
  - multiword units, named entities, phrases, ...
  - morphemes
  - word pairs (analogy tasks)
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- Linguistic annotation
  - word forms (minimally requires tokenisation)
  - often lemmatisation or stemming to reduce data sparseness: go, goes, went, gone, going → go
  - POS disambiguation (light/N vs. light/A vs. light/V)
  - word sense disambiguation (bankriver vs. bankfinance)
  - abstraction: POS tags (or bigrams) as feature terms
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- Trade-off between deeper linguistic analysis and
  - need for language-specific resources
  - possible errors introduced at each stage of the analysis
Effects of linguistic annotation

Nearest neighbours of *walk* (BNC)

<table>
<thead>
<tr>
<th>Word Forms</th>
<th>Lemmatised + POS</th>
</tr>
</thead>
<tbody>
<tr>
<td>stroll</td>
<td>hurry</td>
</tr>
<tr>
<td>walking</td>
<td>stroll</td>
</tr>
<tr>
<td>walked</td>
<td>stride</td>
</tr>
<tr>
<td>go</td>
<td>trudge</td>
</tr>
<tr>
<td>path</td>
<td>amble</td>
</tr>
<tr>
<td>drive</td>
<td>wander</td>
</tr>
<tr>
<td>ride</td>
<td>walk (noun)</td>
</tr>
<tr>
<td>wander</td>
<td>walking</td>
</tr>
<tr>
<td>sprinted</td>
<td>retrace</td>
</tr>
<tr>
<td>sauntered</td>
<td>scuttle</td>
</tr>
</tbody>
</table>

http://clic.cimec.unitn.it/infomap-query/
Effects of linguistic annotation

Nearest neighbours of *arrivare* (Repubblica)

<table>
<thead>
<tr>
<th>word forms</th>
<th>lemmatised + POS</th>
</tr>
</thead>
<tbody>
<tr>
<td>giungere</td>
<td>giungere</td>
</tr>
<tr>
<td>raggiungere</td>
<td>aspettare</td>
</tr>
<tr>
<td>arrivare</td>
<td>attendere</td>
</tr>
<tr>
<td>raggiungimento</td>
<td>arrivo (noun)</td>
</tr>
<tr>
<td>raggiunto</td>
<td>ricevere</td>
</tr>
<tr>
<td>trovare</td>
<td>accontentare</td>
</tr>
<tr>
<td>raggiunge</td>
<td>approdare</td>
</tr>
<tr>
<td>arrivée</td>
<td>pervenire</td>
</tr>
<tr>
<td>concludere</td>
<td>venire</td>
</tr>
<tr>
<td></td>
<td>piombare</td>
</tr>
</tbody>
</table>

http://clic.cimec.unitn.it/infomap-query/
Selection of target and feature terms

- Full-vocabulary models are often unmanageable
  - 762,424 distinct word forms in BNC, 605,910 lemmata
  - large Web corpora have > 10 million distinct word forms
  - low-frequency targets (and features) do not provide reliable distributional information (too much “noise”)
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- Frequency-based selection
  - minimum corpus frequency: \( f \geq F_{\text{min}} \)
  - or accept \( n_w \) most frequent terms
  - sometimes also upper threshold: \( F_{\text{min}} \leq f \leq F_{\text{max}} \)
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  - criterion from IR: document frequency \( df \)
  - terms with high \( df \) are too general ➞ uninformative
  - terms with very low \( df \) may be too sparse to be useful
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- Relevance-based selection
  - criterion from IR: document frequency \( df \)
  - terms with high \( df \) are too general \( \Rightarrow \) uninformative
  - terms with very low \( df \) may be too sparse to be useful

- Other criteria
  - POS-based filter: no function words, only verbs, ...
Overview of DSM parameters

- Pre-processed corpus with linguistic annotation
  - Define target terms
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  - Type & size of co-occurrence

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- Term-term matrix

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  - Probabilistic analysis
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Surface context

Context term occurs within a span of $k$ words around target.

The silhouette of the sun beyond a wide-open bay on the lake; the sun still glitters although evening has arrived in Kuhmo. It’s midsummer; the living room has its instruments and other objects in each of its corners. [L3/R3 span, $k = 6$]

Parameters:

- span size (in words or characters)
- symmetric vs. one-sided span
- uniform or “triangular” (distance-based) weighting
- spans clamped to sentences or other textual units?
## Effect of span size

### Nearest neighbours of *dog* (BNC)

<table>
<thead>
<tr>
<th>2-word span</th>
<th>30-word span</th>
</tr>
</thead>
<tbody>
<tr>
<td>- cat</td>
<td>- kennel</td>
</tr>
<tr>
<td>- horse</td>
<td>- puppy</td>
</tr>
<tr>
<td>- fox</td>
<td>- pet</td>
</tr>
<tr>
<td>- <em>pet</em></td>
<td>- bitch</td>
</tr>
<tr>
<td>- rabbit</td>
<td>- terrier</td>
</tr>
<tr>
<td>- pig</td>
<td>- rottweiler</td>
</tr>
<tr>
<td>- animal</td>
<td>- canine</td>
</tr>
<tr>
<td>- mongrel</td>
<td>- cat</td>
</tr>
<tr>
<td>- sheep</td>
<td>- <em>to bark</em></td>
</tr>
<tr>
<td>- pigeon</td>
<td>- <em>Alsatian</em></td>
</tr>
</tbody>
</table>

[http://clic.cimec.unitn.it/infomap-query/](http://clic.cimec.unitn.it/infomap-query/)
Textual context

Context term is in the same linguistic unit as target.

The silhouette of the sun beyond a wide-open bay on the lake; the sun still glitters although evening has arrived in Kuhmo. It’s midsummer; the living room has its instruments and other objects in each of its corners.

Parameters:

▶ type of linguistic unit
  ▶ sentence
  ▶ paragraph
  ▶ turn in a conversation
  ▶ Web page
Syntactic context

Context term is linked to target by a **syntactic dependency** (e.g. subject, modifier, ...).

The *silhouette* of the *sun* beyond a wide-open *bay* on the lake; the *sun* still *glitters* although evening has arrived in Kuhmo. It’s midsummer; the living room has its instruments and other objects in each of its corners.

Parameters:

- types of syntactic dependency (Padó and Lapata 2007)
- direct vs. indirect dependency paths
  - direct dependencies
  - direct + indirect dependencies
- homogeneous data (e.g. only verb-object) vs. heterogeneous data (e.g. all children and parents of the verb)
- maximal length of dependency path
“Knowledge pattern” context

Context term is linked to target by a lexico-syntactic pattern (text mining, cf. Hearst 1992, Pantel & Pennacchiotti 2008, etc.).

In Provence, Van Gogh painted with bright colors such as red and yellow. These colors produce incredible effects on anybody looking at his paintings.

Parameters:
- inventory of lexical patterns
  - lots of research to identify semantically interesting patterns (cf. Almuhareb & Poesio 2004, Veale & Hao 2008, etc.)
- fixed vs. flexible patterns
  - patterns are mined from large corpora and automatically generalised (optional elements, POS tags or semantic classes)
Structured vs. unstructured context

- In **unstructured** models, context specification acts as a **filter**
  - determines whether context token counts as co-occurrence
  - e.g. must be linked by any syntactic dependency relation
Structured vs. unstructured context

- In **unstructured** models, context specification acts as a **filter**
  - determines whether context token counts as co-occurrence
  - e.g. must be linked by any syntactic dependency relation

- In **structured** models, feature terms are **subtyped**
  - depending on their position in the context
  - e.g. left vs. right context, type of syntactic relation, etc.
## Structured vs. unstructured surface context

A dog bites a man. The man’s dog bites a dog. A dog bites a man.

<table>
<thead>
<tr>
<th>unstructured</th>
<th>bite</th>
</tr>
</thead>
<tbody>
<tr>
<td>dog</td>
<td>4</td>
</tr>
<tr>
<td>man</td>
<td>3</td>
</tr>
</tbody>
</table>
Structured vs. unstructured surface context

A dog bites a man. The man’s dog bites a dog. A dog bites a man.

<table>
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<td>3</td>
</tr>
</tbody>
</table>

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<table>
<thead>
<tr>
<th>structured</th>
<th>bite-l</th>
<th>bite-r</th>
</tr>
</thead>
<tbody>
<tr>
<td>dog</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>man</td>
<td>1</td>
<td>2</td>
</tr>
</tbody>
</table>
### Structured vs. unstructured dependency context

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<table>
<thead>
<tr>
<th><strong>unstructured</strong></th>
<th>bite</th>
</tr>
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<tbody>
<tr>
<td>dog</td>
<td>4</td>
</tr>
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</table>
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**unstructured**

<table>
<thead>
<tr>
<th></th>
<th>bite</th>
</tr>
</thead>
<tbody>
<tr>
<td>dog</td>
<td>4</td>
</tr>
<tr>
<td>man</td>
<td>2</td>
</tr>
</tbody>
</table>

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**structured**

<table>
<thead>
<tr>
<th></th>
<th>bite-subj</th>
<th>bite-obj</th>
</tr>
</thead>
<tbody>
<tr>
<td>dog</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>man</td>
<td>0</td>
<td>2</td>
</tr>
</tbody>
</table>
Comparison

- **Unstructured context**
  - data less sparse (e.g. *man kills* and *kills man* both map to the *kill* dimension of the vector $x_{man}$)

- **Structured context**
  - more sensitive to semantic distinctions (*kill-subj* and *kill-obj* are rather different things!)
  - dependency relations provide a form of syntactic “typing” of the DSM dimensions (the “subject” dimensions, the “recipient” dimensions, etc.)
  - important to account for word-order and compositionality
Overview of DSM parameters

- pre-processed corpus with linguistic annotation
  - term-context matrix
    - define target terms
    - context tokens or types
  - term-term matrix
    - define target & feature terms
    - type & size of co-occurrence

- geometric analysis
- probabilistic analysis
  - embedding learned by neural network
- feature scaling
- similarity/distance measure + normalization
- dimensionality reduction

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Context tokens vs. context types

- Features are usually context tokens, i.e. individual instances
  - document, Wikipedia article, Web page, ...
  - paragraph, sentence, tweet, ...
  - “co-occurrence” count = frequency of term in context token

- Can also be generalised to context types, e.g.
  - type = cluster of near-duplicate documents
  - type = syntactic structure of sentence (ignoring content)
  - type = tweets from same author
  - frequency counts from all instances of type are aggregated

- Context types may be anchored at individual tokens
  - n-gram of words (or POS tags) around target
  - subcategorisation pattern of target verb
  - overlaps with (generalisation of) syntactic co-occurrence
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define target terms
context tokens or types

define target & feature terms
type & size of co-occurrence

M

feature scaling

geometric analysis

probability analysis
embedding learned by neural network

similarity/distance measure + normalization
dimensionality reduction
Marginal and expected frequencies

- Matrix of observed co-occurrence frequencies not sufficient

<table>
<thead>
<tr>
<th>target</th>
<th>feature</th>
<th>$O$</th>
</tr>
</thead>
<tbody>
<tr>
<td>dog</td>
<td>small</td>
<td>855</td>
</tr>
<tr>
<td>dog</td>
<td>domesticated</td>
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- Notation
  - $O = \text{observed co-occurrence frequency}$
Marginal and expected frequencies

- Matrix of observed co-occurrence frequencies not sufficient

<table>
<thead>
<tr>
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<th>feature</th>
<th>$O$</th>
<th>$R$</th>
<th>$C$</th>
</tr>
</thead>
<tbody>
<tr>
<td>dog</td>
<td>small</td>
<td>855</td>
<td>33,338</td>
<td>490,580</td>
</tr>
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<td>918</td>
</tr>
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- Notation
  - $O = \text{observed co-occurrence frequency}$
  - $R = \text{overall frequency of target term} = \text{row marginal frequency}$
  - $C = \text{overall frequency of feature} = \text{column marginal frequency}$
  - $N = \text{sample size} \approx \text{size of corpus}$

Expected co-occurrence frequency

$E = R \cdot \frac{C}{N}$
Marginal and expected frequencies

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- Expected co-occurrence frequency

$$E = \frac{R \cdot C}{N} \quad \longleftrightarrow \quad O$$
Obtaining marginal frequencies

- **Term-document matrix**
  - $R =$ frequency of target term in corpus
  - $C =$ size of document (number of tokens)
  - $N =$ corpus size
Obtaining marginal frequencies

- **Term-document matrix**
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  - \# of dependency instances in which target/feature participates
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  - can be computed from full co-occurrence matrix \( M \)
Obtaining marginal frequencies

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- **Syntactic co-occurrence**
  - \# of dependency instances in which target/feature participates
  - \( N = \) total number of dependency instances
  - can be computed from full co-occurrence matrix \( M \)

- **Textual co-occurrence**
  - \( R, C, O \) are “document” frequencies, i.e. number of context units in which target, feature or combination occurs
  - \( N = \) total \# of context units
Obtaining marginal frequencies

- **Surface co-occurrence**
  - it is quite tricky to obtain fully consistent counts (Evert 2008)
  - at least correct $E$ for span size $k$ (= number of tokens in span)

\[
E = k \cdot \frac{R \cdot C}{N}
\]

with $R, C =$ individual corpus frequencies and $N =$ corpus size

- can also be implemented by pre-multiplying $R' = k \cdot R$

alternatively, compute marginals and sample size by summing over full co-occurrence matrix ($\Rightarrow E$ as above, but inflated $N$)
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  - alternatively, compute marginals and sample size by summing over full co-occurrence matrix (→ $E$ as above, but inflated $N$)

- NB: shifted PPMI (Levy and Goldberg 2014) corresponds to a post-hoc application of the span size adjustment
  - performs worse than PPMI, but paper suggests they already approximate correct $E$ by summing over co-occurrence matrix
Marginal frequencies in *wordspace*

DSM objects in *wordspace* include marginal frequencies as well as counts of nonzero cells for rows and columns.

```r
> TT$rows
   term f nnzero
1  cat 22007  5
2  dog  50807  7
3 animal  77053  7
4  time 1156693  7
5 reason  95047  6
6  cause  54739  5
7 effect 133102  6
> TT$cols
...
> TT$globals$N
[1] 199902178
> TT$M  # the full co-occurrence matrix
```
Geometric vs. probabilistic interpretation

- Geometric interpretation
  - row vectors as points or arrows in $n$-dimensional space
  - very intuitive, good for visualisation
  - use techniques from geometry and matrix algebra

- Probabilistic interpretation
  - co-occurrence matrix as observed sample statistic that is “explained” by a generative probabilistic model
  - e.g. probabilistic LSA (Hoffmann 1999), Latent Semantic Clustering (Rooth et al. 1999), Latent Dirichlet Allocation (Blei et al. 2003), etc.
  - explicitly accounts for random variation of frequency counts
  - recent work: neural word embeddings
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* focus on geometric interpretation in this tutorial
Overview of DSM parameters

1. Pre-processed corpus with linguistic annotation
   - Define target terms
   - Context tokens or types
   - Type & size of co-occurrence

2. Term-context matrix
   - Geometric analysis
   - Feature scaling
   - Similarity/distance measure + normalization
   - Dimensionality reduction

3. Term-term matrix
   - Probabilistic analysis
   - Embedding learned by neural network
Feature scaling

Feature scaling is used to “discount” less important features:

- Logarithmic scaling: \( O' = \log(O + 1) \)
  (cf. Weber-Fechner law for human perception)

Statistical association measures (Evert 2004, 2008) take frequency of target term and feature into account

often based on comparison of observed and expected co-occurrence frequency

measures differ in how they balance \( O \) and \( E \)
Feature scaling

Feature scaling is used to “discount” less important features:

- **Logarithmic scaling**: $O' = \log(O + 1)$
  (cf. Weber-Fechner law for human perception)
- **Relevance weighting**, e.g. $tf.idf$ (information retrieval)

$$tf.idf = tf \cdot \log(D/df)$$

- $tf = \text{co-occurrence frequency } O$
- $df = \text{document frequency of feature (or nonzero count)}$
- $D = \text{total number of documents (or row count of } M)$
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- Statistical **association measures** (Evert 2004, 2008) take frequency of target term and feature into account
  - often based on comparison of observed and expected co-occurrence frequency
  - measures differ in how they balance $O$ and $E$
Simple association measures

<table>
<thead>
<tr>
<th>target</th>
<th>feature</th>
<th>$O$</th>
<th>$E$</th>
</tr>
</thead>
<tbody>
<tr>
<td>dog</td>
<td>small</td>
<td>855</td>
<td>134.34</td>
</tr>
<tr>
<td>dog</td>
<td>domesticated</td>
<td>29</td>
<td>0.25</td>
</tr>
<tr>
<td>dog</td>
<td>sgjkl</td>
<td>1</td>
<td>0.00027</td>
</tr>
</tbody>
</table>
Simple association measures

- pointwise Mutual Information (MI)

\[
MI = \log_2 \frac{O}{E}
\]

<table>
<thead>
<tr>
<th>target</th>
<th>feature</th>
<th>O</th>
<th>E</th>
<th>MI</th>
</tr>
</thead>
<tbody>
<tr>
<td>dog</td>
<td>small</td>
<td>855</td>
<td>134.34</td>
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Simple association measures

- **pointwise Mutual Information** (MI)
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- **local MI**
  \[ \text{local-MI} = O \cdot \text{MI} = O \cdot \log_2 \frac{O}{E} \]

<table>
<thead>
<tr>
<th>target</th>
<th>feature</th>
<th>$O$</th>
<th>$E$</th>
<th>MI</th>
<th>local-MI</th>
</tr>
</thead>
<tbody>
<tr>
<td>dog</td>
<td>small</td>
<td>855</td>
<td>134.34</td>
<td>2.67</td>
<td>2282.88</td>
</tr>
<tr>
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<td>0.25</td>
<td>6.85</td>
<td>198.76</td>
</tr>
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  \[ \text{local-MI} = O \cdot \text{MI} = O \cdot \log_2 \frac{O}{E} \]

- **t-score**
  \[ t = \frac{O - E}{\sqrt{O}} \]

<table>
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<tr>
<th>target</th>
<th>feature</th>
<th>O</th>
<th>E</th>
<th>MI</th>
<th>local-MI</th>
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Other association measures

- simple log-likelihood (≈ local-MI)

\[ G^2 = \pm 2 \cdot \left( O \cdot \log_2 \frac{O}{E} - (O - E) \right) \]

with positive sign for \( O > E \) and negative sign for \( O < E \)
Other association measures

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$$G^2 = \pm 2 \cdot \left( O \cdot \log_2 \frac{O}{E} - (O - E) \right)$$

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$$\text{Dice} = \frac{2O}{R + C}$$
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- **Dice coefficient**

\[ \text{Dice} = \frac{2O}{R + C} \]

- Many other simple association measures (AMs) available
- **Further AMs computed from full contingency tables**, see
  - Evert (2008)
  - [http://www.collocations.de/](http://www.collocations.de/)
  - [http://sigil.r-forge.r-project.org/](http://sigil.r-forge.r-project.org/)
Applying association scores in wordspace

```r
> options(digits=3)  # print fractional values with limited precision
> dsm.score(TT, score="MI", sparse=FALSE, matrix=TRUE)

<table>
<thead>
<tr>
<th></th>
<th>breed</th>
<th>tail</th>
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<th>explain</th>
<th>likely</th>
</tr>
</thead>
<tbody>
<tr>
<td>cat</td>
<td>6.21</td>
<td>4.568</td>
<td>3.129</td>
<td>2.801</td>
<td>-Inf</td>
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<td>dog</td>
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<td>2.132</td>
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<tr>
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The sparseness of the matrix has been lost! Cells with score $x = -\infty$ are inconvenient. The distribution of scores may be even more skewed than co-occurrence frequencies (esp. for local-MI).
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Sparse association measures

- Sparse association scores are cut off at zero, i.e.

\[ f(x) = \begin{cases} 
  x & x > 0 \\
  0 & x \leq 0 
\end{cases} \]

- Also known as “positive” scores
  - **PPMI** = positive pointwise MI (e.g. Bullinaria and Levy 2007)
  - **wordspace** computes sparse AMs by default ➔ "MI" = PPMI
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- Also known as “positive” scores
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  - wordspace computes sparse AMs by default \(\Rightarrow\) "MI" = PPMI

- Preserves sparseness if \(x \leq 0\) for all empty cells \((O = 0)\)
  - sparseness may even increase: cells with \(x < 0\) become empty

- Usually combined with signed association measure satisfying
  - \(x > 0\) for \(O > E\)
  - \(x < 0\) for \(O < E\)
Score transformations

An additional scale transformation can be applied in order to de-skew association scores:

- signed logarithmic transformation
  \[ f(x) = \pm \log(|x| + 1) \]
- sigmoid transformation as soft binarization
  \[ f(x) = \tanh(x) \]
- sparse AM as cutoff transformation
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▶ signed logarithmic transformation

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  $$f(x) = \tanh x$$

- **sparse AM** as cutoff transformation
Association scores & transformations in *wordspace*

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> dsm.score(TT, score="MI", matrix=TRUE)  # PPMI

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<td>3.92</td>
<td>2.32</td>
<td>0.000</td>
<td>0.0000</td>
</tr>
<tr>
<td>animal</td>
<td>3.50</td>
<td>2.13</td>
<td>4.75</td>
<td>2.83</td>
<td>0.000</td>
<td>0.0000</td>
</tr>
<tr>
<td>time</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.000</td>
<td>0.639</td>
</tr>
<tr>
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<td>0.791</td>
<td>1.6312</td>
</tr>
</tbody>
</table>

> dsm.score(TT, score="simple-ll", matrix=TRUE)

> dsm.score(TT, score="simple-ll", transf="log", matrix=T)  # logarithmic co-occurrence frequency

> dsm.score(TT, score="freq", transform="log", matrix=T)

# now try other parameter combinations

> ?dsm.score  # read help page for available parameter settings
```
Scaling of column vectors

- In statistical analysis and machine learning, features are usually **centred** and **scaled** so that

\[
\begin{align*}
\text{mean} & \quad \mu = 0 \\
\text{variance} & \quad \sigma^2 = 1
\end{align*}
\]
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  \end{align*}
  \]

- In DSM research, this step is less common for columns of \(\mathbf{M}\)
  - centring is a prerequisite for certain dimensionality reduction and data analysis techniques (esp. PCA)
  - but co-occurrence matrix no longer sparse!
  - scaling may give too much weight to rare features
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- centring is a prerequisite for certain dimensionality reduction and data analysis techniques (esp. PCA)
- but co-occurrence matrix no longer sparse!
- scaling may give too much weight to rare features

$M$ cannot be row-normalised and column-scaled at the same time (result depends on ordering of the two steps)
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1. Pre-processed corpus with linguistic annotation
   - Define target terms
   - Context tokens or types
   - Define target & feature terms
   - Type & size of co-occurrence
   - Feature scaling
   - Similarity/distance measure + normalization
   - Dimensionality reduction
   - Geometric analysis
   - M
   - Probabilistic analysis
   - Embedding learned by neural network
Geometric distance = metric

- **Distance** between vectors \( u, v \in \mathbb{R}^n \) \( \Rightarrow \) (dis)similarity
  - \( u = (u_1, \ldots, u_n) \)
  - \( v = (v_1, \ldots, v_n) \)

\[
\begin{align*}
\| \mathbf{u} - \mathbf{v} \|_2 &= \sqrt{(u_1 - v_1)^2 + \cdots + (u_n - v_n)^2} \\
\| \mathbf{u} - \mathbf{v} \|_1 &= |u_1 - v_1| + \cdots + |u_n - v_n|
\end{align*}
\]

Both are special cases of the Minkowski \( p \)-distance

\( d_p(\mathbf{u}, \mathbf{v}) = \left( \sum_{i=1}^{n} |u_i - v_i|^p \right)^{1/p} \)

(\text{for} \( p \in [1, \infty) \))
**Geometric distance = metric**

- **Distance** between vectors $\mathbf{u}, \mathbf{v} \in \mathbb{R}^n \Rightarrow (\text{dis})\text{similarity}
  - $\mathbf{u} = (u_1, \ldots, u_n)$
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- **Euclidean** distance $d_2(\mathbf{u}, \mathbf{v})$

\[
d_2(\mathbf{u}, \mathbf{v}) := \sqrt{(u_1 - v_1)^2 + \cdots + (u_n - v_n)^2}
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- “City block” **Manhattan** distance $d_1(\mathbf{u}, \mathbf{v})$

\[
d_1(\mathbf{u}, \mathbf{v}) := |u_1 - v_1| + \cdots + |u_n - v_n|
\]
Geometric distance = metric

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  (for $p \in [1, \infty]$)

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- "City block" **Manhattan** distance \( d_1(u, v) \)
- Both are special cases of the **Minkowski** \( p \)-distance \( d_p(u, v) \) (for \( p \in [1, \infty] \))

\[
d_p(u, v) := (|u_1 - v_1|^p + \cdots + |u_n - v_n|^p)^{1/p}
\]

\[
d_\infty(u, v) = \max\{|u_1 - v_1|, \ldots, |u_n - v_n|\}
\]
Geometric distance = metric

- **Distance** between vectors \( \mathbf{u}, \mathbf{v} \in \mathbb{R}^n \rightarrow \) (dis)similarity
  - \( \mathbf{u} = (u_1, \ldots, u_n) \)
  - \( \mathbf{v} = (v_1, \ldots, v_n) \)
- **Euclidean** distance \( d_2(\mathbf{u}, \mathbf{v}) \)
- “City block” **Manhattan** distance \( d_1(\mathbf{u}, \mathbf{v}) \)
- Extension of \( p \)-distance \( d_p(\mathbf{u}, \mathbf{v}) \) (for \( 0 \leq p \leq 1 \))

\[
d_p(\mathbf{u}, \mathbf{v}) := |u_1 - v_1|^p + \cdots + |u_n - v_n|^p
\]

\[
d_0(\mathbf{u}, \mathbf{v}) = \#\{i \mid u_i \neq v_i\}
\]
Computing distances

Preparation: store “scored” matrix in DSM object

```r
> TT <- dsm.score(TT, score="freq", transform="log")
```
Computing distances

Preparation: store “scored” matrix in DSM object

```r
> TT <- dsm.score(TT, score="freq", transform="log")
```

Compute distances between individual term pairs ...

```r
> pair.distances(c("cat","cause"), c("animal","effect"), TT, method="euclidean")

cat/animal cause/effect
4.16 1.53
```
Computing distances

Preparation: store “scored” matrix in DSM object

```r
> TT <- dsm.score(TT, score="freq", transform="log")
```

Compute distances between individual term pairs...

```r
> pair.distances(c("cat","cause"), c("animal","effect"), TT, method="euclidean")

  cat/animal  cause/effect
  4.16       1.53
```

... or full distance matrix.

```r
> dist.matrix(TT, method="euclidean")
> dist.matrix(TT, method="minkowski", p=4)
```
Distance and vector length = norm

- Intuitively, distance $d(u, v)$ should correspond to length $\|u - v\|$ of displacement vector $u - v$
  - $d(u, v)$ is a metric
  - $\|u - v\|$ is a norm
  - $\|u\| = d(u, 0)$
**Distance and vector length = norm**

- Intuitively, distance \( d(u, v) \) should correspond to length \( \|u - v\| \) of displacement vector \( u - v \)
  - \( d(u, v) \) is a **metric**
  - \( \|u - v\| \) is a **norm**
  - \( \|u\| = d(u, 0) \)
- Such a metric is always **translation-invariant**
Distance and vector length = norm

- Intuitively, distance \(d(u, v)\) should correspond to length \(\|u - v\|\) of displacement vector \(u - v\)
  - \(d(u, v)\) is a metric
  - \(\|u - v\|\) is a norm
  - \(\|u\| = d(u, 0)\)

- Such a metric is always translation-invariant

- \(d_p(u, v) = \|u - v\|_p\)

- **Minkowski \(p\)-norm** for \(p \in [1, \infty)\) (not \(p < 1\)):
  \[
  \|u\|_p := (|u_1|^p + \cdots + |u_n|^p)^{1/p}
  \]
Normalisation of row vectors

- Geometric distances only meaningful for vectors of the same length $\|x\|$

Two dimensions of English V–Obj DSM

- $\alpha = 54.3^\circ$

- The diagram shows the positions of words 'cat', 'dog', 'knife', and 'boat' in a two-dimensional DSM space.

- The angle $\alpha$ between the vectors for 'cat' and 'dog' is illustrated.

Norm must be compatible with distance measure!
Normalisation of row vectors

- Geometric distances only meaningful for vectors of the same length $\|x\|$

- Normalize by scalar division: $x' = x/\|x\| = (\frac{x_1}{\|x\|}, \frac{x_2}{\|x\|}, \ldots)$ with $\|x'\| = 1$
Normalisation of row vectors

- Geometric distances only meaningful for vectors of the same length $\|x\|$
- Normalize by scalar division: $x' = x / \|x\| = (\frac{x_1}{\|x\|}, \frac{x_2}{\|x\|}, \ldots)$ with $\|x'\| = 1$
- Norm must be compatible with distance measure!
- Special case: scale to relative frequencies with $\|x\|_1 = |x_1| + \cdots + |x_n|$ → probabilistic interpretation
## Norms and normalization

```r
> rowNorms(TT$S, method="euclidean")
cat   dog   animal  time  reason  cause  effect
6.90  8.96  8.82    10.29  8.13   6.86   6.52
```

```r
> TT <- dsm.score(TT, score="freq", transform="log", normalize=TRUE, method="euclidean")
> rowNorms(TT$S, method="euclidean")  # all = 1 now
> dist.matrix(TT, method="euclidean")
cat   dog   animal  time  reason  cause  effect  
cat 0.000 0.224 0.473 0.782 1.121 1.239 1.161
dog 0.224 0.000 0.398 0.698 1.065 1.179 1.113
animal 0.473 0.398 0.000 0.426 0.841 0.971 0.860
time 0.782 0.698 0.426 0.000 0.475 0.585 0.502
reason 1.121 1.065 0.841 0.475 0.000 0.277 0.198
cause 1.239 1.179 0.971 0.585 0.277 0.000 0.224
effect 1.161 1.113 0.860 0.502 0.198 0.224 0.000
```
Other distance measures

- Information theory: **Kullback-Leibler (KL) divergence** for probability vectors (non-negative, $\|x\|_1 = 1$)

$$D(u\|v) = \sum_{i=1}^{n} u_i \cdot \log_2 \frac{u_i}{v_i}$$
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- Properties of KL divergence
  - most appropriate in a probabilistic interpretation of $M$
  - zeroes in $v$ without corresponding zeroes in $u$ are problematic
  - not symmetric, unlike geometric distance measures
  - alternatives: skew divergence, Jensen-Shannon divergence
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- A symmetric distance measure (Endres and Schindelin 2003)

$$D_{uv} = D(u\|z) + D(v\|z) \quad \text{with} \quad z = \frac{u + v}{2}$$
Similarity measures

- Angle $\alpha$ between vectors $\mathbf{u}, \mathbf{v} \in \mathbb{R}^n$ is given by

$$\cos \alpha = \frac{\sum_{i=1}^{n} u_i \cdot v_i}{\sqrt{\sum_i u_i^2} \cdot \sqrt{\sum_i v_i^2}}$$

$$= \frac{\mathbf{u}^T \mathbf{v}}{\| \mathbf{u} \|_2 \cdot \| \mathbf{v} \|_2}$$

Two dimensions of English V–Obj DSM

$\alpha = 54.3^\circ$
Similarity measures

- **Angle** $\alpha$ between vectors $\mathbf{u}, \mathbf{v} \in \mathbb{R}^n$ is given by

$$\cos \alpha = \frac{\sum_{i=1}^{n} u_i \cdot v_i}{\sqrt{\sum_i u_i^2} \cdot \sqrt{\sum_i v_i^2}} = \frac{\mathbf{u}^T \mathbf{v}}{\|\mathbf{u}\|_2 \cdot \|\mathbf{v}\|_2}$$

- **cosine** measure of similarity: $\cos \alpha$
  - $\cos \alpha = 1 \rightarrow$ collinear
  - $\cos \alpha = 0 \rightarrow$ orthogonal

- **Corresponding metric:** angular distance $\alpha$

![Diagram showing two dimensions of English V-Obj DSM](image-url)
Overview of DSM parameters

- pre-processed corpus with linguistic annotation
  - define target terms
  - define target & feature terms
  - context tokens or types
  - type & size of co-occurrence
  - term-context matrix
  - term-term matrix
  - geometric analysis
    - feature scaling
    - similarity/distance measure + normalization
    - dimensionality reduction
  - probabilistic analysis
    - embedding learned by neural network
Dimensionality reduction $\equiv$ model compression

- Co-occurrence matrix $\mathbf{M}$ is often unmanageably large and can be extremely sparse
  - Google Web1T5: $1\text{M} \times 1\text{M}$ matrix with one trillion cells, of which less than 0.05% contain nonzero counts (Evert 2010)
- Compress matrix by reducing dimensionality ($\equiv$ rows)
Dimensionality reduction $\Rightarrow$ model compression

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- Feature selection: columns with high frequency & variance
  - measured by entropy, chi-squared test, nonzero count, ... 
  - may select similar dimensions and discard valuable information
  - joint selection of multiple features is useful but expensive
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- **Projection** into (linear) subspace
  - principal component analysis (PCA)
  - independent component analysis (ICA)
  - random indexing (RI)
  - intuition: preserve distances between data points
Dimensionality reduction & latent dimensions

Landauer and Dumais (1997) claim that LSA dimensionality reduction (and related PCA technique) uncovers **latent dimensions** by exploiting correlations between features.

- **Example**: term-term matrix
- **V-Obj cooc’s extracted from BNC**
  - targets = noun lemmas
  - features = verb lemmas
- **feature scaling**: association scores (modified log Dice coefficient)
- **$k = 111$ nouns with $f \geq 20$**
  (must have non-zero row vectors)
- **$n = 2$ dimensions**: buy and sell

<table>
<thead>
<tr>
<th>noun</th>
<th>buy</th>
<th>sell</th>
</tr>
</thead>
<tbody>
<tr>
<td>bond</td>
<td>0.28</td>
<td>0.77</td>
</tr>
<tr>
<td>cigarette</td>
<td>-0.52</td>
<td>0.44</td>
</tr>
<tr>
<td>dress</td>
<td>0.51</td>
<td>-1.30</td>
</tr>
<tr>
<td>freehold</td>
<td>-0.01</td>
<td>-0.08</td>
</tr>
<tr>
<td>land</td>
<td>1.13</td>
<td>1.54</td>
</tr>
<tr>
<td>number</td>
<td>-1.05</td>
<td>-1.02</td>
</tr>
<tr>
<td>per</td>
<td>-0.35</td>
<td>-0.16</td>
</tr>
<tr>
<td>pub</td>
<td>-0.08</td>
<td>-1.30</td>
</tr>
<tr>
<td>share</td>
<td>1.92</td>
<td>1.99</td>
</tr>
<tr>
<td>system</td>
<td>-1.63</td>
<td>-0.70</td>
</tr>
</tbody>
</table>
Dimensionality reduction & latent dimensions
Motivating latent dimensions & subspace projection

- The **latent property** of being a commodity is “expressed” through associations with several verbs: *sell, buy, acquire, ...*

- Consequence: these DSM dimensions will be **correlated**
Motivating latent dimensions & subspace projection

- The **latent property** of being a commodity is “expressed” through associations with several verbs: *sell*, *buy*, *acquire*, ...
- Consequence: these DSM dimensions will be **correlated**

- Identify **latent dimension** by looking for strong correlations (or weaker correlations between large sets of features)
- Projection into subspace $V$ of $k < n$ latent dimensions as a “**noise reduction**” technique $\rightarrow$ LSA
- Assumptions of this approach:
  - “latent” distances in $V$ are semantically meaningful
  - other “residual” dimensions represent chance co-occurrence patterns, often particular to the corpus underlying the DSM
Centering the data set

- **Uncentered data set**
- **Centered data set**
- **Variance of centered data**
Centering the data set

- Uncentered data set
- **Centered** data set
- Variance of centered data
Centering the data set

- Uncentered data set
- Centered data set
- Variance of centered data

\[ \sigma^2 = \frac{1}{k-1} \sum_{i=1}^{k} \| \mathbf{x}^{(i)} \|^2 \]
Projection and preserved variance: examples
Projection and preserved variance: examples
Projection and preserved variance: examples
Projection and preserved variance: examples

\[ \text{variance} = 0.72 \]
Projection and preserved variance: examples
Projection and preserved variance: examples

\[
\text{variance} = 0.9
\]
Orthogonal PCA dimensions
Dimensionality reduction in practice

# it is customary to omit the centring: SVD dimensionality reduction
> TT2 <- dsm.projection(TT, n=2, method="svd")
> TT2

<table>
<thead>
<tr>
<th></th>
<th>svd1</th>
<th>svd2</th>
</tr>
</thead>
<tbody>
<tr>
<td>cat</td>
<td>-0.733</td>
<td>-0.6615</td>
</tr>
<tr>
<td>dog</td>
<td>-0.782</td>
<td>-0.6110</td>
</tr>
<tr>
<td>animal</td>
<td>-0.914</td>
<td>-0.3606</td>
</tr>
<tr>
<td>time</td>
<td>-0.993</td>
<td>0.0302</td>
</tr>
<tr>
<td>reason</td>
<td>-0.889</td>
<td>0.4339</td>
</tr>
<tr>
<td>cause</td>
<td>-0.817</td>
<td>0.5615</td>
</tr>
<tr>
<td>effect</td>
<td>-0.871</td>
<td>0.4794</td>
</tr>
</tbody>
</table>

> x <- TT2[, 1]  # first latent dimension
> y <- TT2[, 2]  # second latent dimension
> plot(TT2, pch=20, col="red",
       xlim=extendrange(x), ylim=extendrange(y))
> text(TT2, rownames(TT2), pos=3)
Outline

DSM parameters
  A taxonomy of DSM parameters
  Examples

Building a DSM
  Sparse matrices
  Example: a verb-object DSM
Some well-known DSM examples

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<tr>
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<tr>
<td>- weighting: log term frequency and term entropy</td>
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<td>- distance measure: cosine</td>
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**Latent Semantic Analysis (Landauer and Dumais 1997)**
- term-context matrix with document context
- weighting: log term frequency and term entropy
- distance measure: cosine
- dimensionality reduction: SVD

**Hyperspace Analogue to Language (Lund and Burgess 1996)**
- term-term matrix with surface context
- structured (left/right) and distance-weighted frequency counts
- distance measure: Minkowski metric \(1 \leq p \leq 2\)
- dimensionality reduction: feature selection (high variance)
Some well-known DSM examples

**Infomap NLP (Widdows 2004)**

- term-term matrix with unstructured surface context
- weighting: none
- distance measure: cosine
- dimensionality reduction: SVD
Some well-known DSM examples

Infomap NLP (Widdows 2004)

- term-term matrix with unstructured surface context
- weighting: none
- distance measure: cosine
- dimensionality reduction: SVD

Random Indexing (Karlgren and Sahlgren 2001)

- term-term matrix with unstructured surface context
- weighting: various methods
- distance measure: various methods
- dimensionality reduction: random indexing (RI)
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<table>
<thead>
<tr>
<th>Dependency Vectors (Padó and Lapata 2007)</th>
</tr>
</thead>
<tbody>
<tr>
<td>▶ term-term matrix with unstructured dependency context</td>
</tr>
<tr>
<td>▶ weighting: log-likelihood ratio</td>
</tr>
<tr>
<td>▶ distance measure: PPMI-weighted Dice (Lin 1998)</td>
</tr>
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<td>▶ dimensionality reduction: none</td>
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<td>- dimensionality reduction: none</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Distributional Memory (Baroni and Lenci 2010)</th>
</tr>
</thead>
<tbody>
<tr>
<td>- term-term matrix with structured and unstructured dependencies + knowledge patterns</td>
</tr>
<tr>
<td>- weighting: local-MI on type frequencies of link patterns</td>
</tr>
<tr>
<td>- distance measure: cosine</td>
</tr>
<tr>
<td>- dimensionality reduction: none</td>
</tr>
</tbody>
</table>
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Scaling up to the real world

- So far, we have worked on minuscule toy models
- We want to scale up to real world data sets now
Scaling up to the real world

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  - We want to scale up to real world data sets now

- Example 1: window-based DSM on BNC content words
  - 83,926 lemma types with \( f \geq 10 \)
  - term-term matrix with \( 83,926 \cdot 83,926 = 7 \) billion entries
  - standard representation requires 56 GB of RAM (8-byte floats)
  - only 22.1 million non-zero entries (\( \approx 0.32\% \))
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✍ We want to scale up to real world data sets now

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  - standard representation requires 56 GB of RAM (8-byte floats)
  - only 22.1 million non-zero entries (\( = 0.32\% \))

- Example 2: Google Web 1T 5-grams (1 trillion words)
  - more than 1 million word types with \( f \geq 2500 \)
  - term-term matrix with 1 trillion entries requires 8 TB RAM
  - only 400 million non-zero entries (\( = 0.04\% \))
### Sparse matrix representation

- **Invented example of a *sparsely populated* DSM matrix**

<table>
<thead>
<tr>
<th></th>
<th>eat</th>
<th>get</th>
<th>hear</th>
<th>kill</th>
<th>see</th>
<th>use</th>
</tr>
</thead>
<tbody>
<tr>
<td>boat</td>
<td>.</td>
<td>59</td>
<td>.</td>
<td>.</td>
<td>39</td>
<td>23</td>
</tr>
<tr>
<td>cat</td>
<td>.</td>
<td>.</td>
<td>.</td>
<td>26</td>
<td>58</td>
<td>.</td>
</tr>
<tr>
<td>cup</td>
<td>.</td>
<td>98</td>
<td>.</td>
<td>.</td>
<td>.</td>
<td>.</td>
</tr>
<tr>
<td>dog</td>
<td>33</td>
<td>.</td>
<td>42</td>
<td>.</td>
<td>83</td>
<td>.</td>
</tr>
<tr>
<td>knife</td>
<td>.</td>
<td>.</td>
<td>.</td>
<td>.</td>
<td>.</td>
<td>84</td>
</tr>
<tr>
<td>pig</td>
<td>9</td>
<td>.</td>
<td>27</td>
<td>.</td>
<td>.</td>
<td>.</td>
</tr>
</tbody>
</table>

Store only non-zero entries in compact sparse matrix format:

<table>
<thead>
<tr>
<th>row</th>
<th>col</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>59</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
<td>39</td>
</tr>
<tr>
<td>1</td>
<td>4</td>
<td>23</td>
</tr>
<tr>
<td>1</td>
<td>5</td>
<td>39</td>
</tr>
<tr>
<td>1</td>
<td>6</td>
<td>58</td>
</tr>
<tr>
<td>1</td>
<td>7</td>
<td>84</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>26</td>
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<td>3</td>
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<td>98</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
<td>42</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
<td>83</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>84</td>
</tr>
</tbody>
</table>
Sparse matrix representation

- **Invented example of a sparsely populated DSM matrix**

<table>
<thead>
<tr>
<th></th>
<th>eat</th>
<th>get</th>
<th>hear</th>
<th>kill</th>
<th>see</th>
<th>use</th>
</tr>
</thead>
<tbody>
<tr>
<td>boat</td>
<td></td>
<td>59</td>
<td></td>
<td>39</td>
<td>23</td>
<td></td>
</tr>
<tr>
<td>cat</td>
<td>.</td>
<td>.</td>
<td>.</td>
<td>26</td>
<td>58</td>
<td>.</td>
</tr>
<tr>
<td>cup</td>
<td>.</td>
<td>98</td>
<td>.</td>
<td>.</td>
<td>.</td>
<td>.</td>
</tr>
<tr>
<td>dog</td>
<td>33</td>
<td>.</td>
<td>42</td>
<td>.</td>
<td>83</td>
<td>.</td>
</tr>
<tr>
<td>knife</td>
<td>.</td>
<td>.</td>
<td>.</td>
<td>.</td>
<td>.</td>
<td>84</td>
</tr>
<tr>
<td>pig</td>
<td>9</td>
<td>.</td>
<td>.</td>
<td>27</td>
<td>.</td>
<td>.</td>
</tr>
</tbody>
</table>

- **Store only non-zero entries in compact sparse matrix format**

<table>
<thead>
<tr>
<th>row</th>
<th>col</th>
<th>value</th>
<th>row</th>
<th>col</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>59</td>
<td>4</td>
<td>1</td>
<td>33</td>
</tr>
<tr>
<td>1</td>
<td>5</td>
<td>39</td>
<td>4</td>
<td>3</td>
<td>42</td>
</tr>
<tr>
<td>1</td>
<td>6</td>
<td>23</td>
<td>4</td>
<td>5</td>
<td>83</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>26</td>
<td>5</td>
<td>6</td>
<td>84</td>
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<tr>
<td>2</td>
<td>5</td>
<td>58</td>
<td>6</td>
<td>1</td>
<td>9</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>98</td>
<td>6</td>
<td>4</td>
<td>27</td>
</tr>
</tbody>
</table>
Working with sparse matrices

- Compressed format: each row index (or column index) stored only once, followed by non-zero entries in this row (or column)
  - convention: **column-major** matrix (data stored by columns)

- Specialised algorithms for sparse matrix algebra
  - especially matrix multiplication, solving linear systems, etc.
  - take care to avoid operations that create a dense matrix!
Working with sparse matrices

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- R implementation: Matrix package
  - essential for real-life distributional semantics
  - wordspace provides additional support for sparse matrices (vector distances, sparse SVD, ...)

- Other software: Matlab, Octave, Python + SciPy
Outline

DSM parameters
A taxonomy of DSM parameters
Examples

Building a DSM
Sparse matrices
Example: a verb-object DSM
**Triplet tables**

- A sparse DSM matrix can be represented as a table of triplets (target, feature, co-occurrence frequency)
  - for syntactic co-occurrence and term-document matrices, marginals can be computed from a complete triplet table
  - for surface and textual co-occurrence, marginals have to be provided in separate files (see `?read.dsm.triplet`)

<table>
<thead>
<tr>
<th>noun</th>
<th>rel</th>
<th>verb</th>
<th>f</th>
<th>mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>dog</td>
<td>subj</td>
<td>bite</td>
<td>3</td>
<td>spoken</td>
</tr>
<tr>
<td>dog</td>
<td>subj</td>
<td>bite</td>
<td>12</td>
<td>written</td>
</tr>
<tr>
<td>dog</td>
<td>obj</td>
<td>bite</td>
<td>4</td>
<td>written</td>
</tr>
<tr>
<td>dog</td>
<td>obj</td>
<td>stroke</td>
<td>3</td>
<td>written</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td></td>
</tr>
</tbody>
</table>

- DSM_VerbNounTriples_BNC contains additional information
  - syntactic relation between noun and verb
  - written or spoken part of the British National Corpus
Constructing a DSM from a triplet table

- Additional information can be used for filtering (verb-object relation), or aggregate frequencies (spoken + written BNC)

> tri <- subset(DSM_VerbNounTriples_BNC, rel == "obj")

- Construct DSM object from triplet input
  - raw.freq=TRUE indicates raw co-occurrence frequencies (rather than a pre-weighted DSM)
  - constructor aggregates counts from duplicate entries
  - marginal frequencies are automatically computed

> VObj <- dsm(target=tri$noun, feature=tri$verb, score=tri$f, raw.freq=TRUE)
> VObj  # inspect marginal frequencies (e.g. head(VObj$rows, 20))
Exploring the DSM

```r
> VObj <- dsm.score(VObj, score="MI", normalize=TRUE)

> nearest.neighbours(VObj, "dog") # angular distance

    horse  cat animal  rabbit    fish  guy
cichlid 73.9  75.9  76.2  77.0  77.2  78.5
     kid 78.6  79.0  79.1  79.5

> nearest.neighbours(VObj, "dog", method="manhattan")
# NB: we used an incompatible Euclidean normalization!

> VObj50 <- dsm.projection(VObj, n=50, method="svd")
> nearest.neighbours(VObj50, "dog")
```


Evert, Stefan (2010). Google Web 1T5 n-grams made easy (but not for the computer). In *Proceedings of the 6th Web as Corpus Workshop (WAC-6)*, pages 32–40, Los Angeles, CA.
References II


References III
